Dynamic response of flexible strip-foundations by boundary and finite elements

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A time domain Boundary Element-Finite method is employed to determine the dynamic response of flexible surface two-dimensional foundations under conditions of plane strain placed on an elastic soil medium and subjected either to transient external forces or to obliquely incident seismic waves. The elastic, isotropic, and homogeneous soil medium is treated by the time domain Direct Boundary Element Method, while the flexible foundation is treated by the Finite Element Method. The two methods are appropriately combined through equilibrium and compatibility considerations at the soil-foundation interface. Parametric studies examining the effect of the relative stiffness between the foundation and the soil and the spatial distribution of the dynamic disturbances on the foundation response are presented.

INTRODUCTION

The dynamic analysis of foundations placed on an elastic soil medium has received considerable attention in recent years. In the great majority of cases, the assumption is made that the foundation is rigid. An almost complete review of the research pertinent to the dynamic analysis of rigid three-dimensional (3-D) and two-dimensional (2-D) foundations on an elastic soil can be found in Karabalis and Beskos[1] and Spyrakos and Beskos[2], respectively. However, the rigid foundation assumption which appears to be reasonable for massive structures such as nuclear power plants may be inappropriate for many other structures such as buildings with large plane dimensions on a mat foundation, buildings with a stiff central core combined with flexible frames on a mat foundation, concrete gravity dams, and earth dams[3,4]. Goschy[5,6], in his work on soil-structure interaction of tall large-panel buildings, mentions that field observations have proven that foundations of such buildings can be considered infinitely stiff in the shorter and flexible in the longer direction. On the basis of full-scale measurements on a particular structure, Wong, Luco and Trifunac[7] found that the rigid mat assumption leads to acceptable results describing the overall motion of the superstructure, but does not represent properly deformations close to the foundation level. This paper deals with the determination of the dynamic response of flexible 2-D surface foundations.

While the effect of mat flexibility has been extensively treated under static conditions[8], such a treatment for the dynamic case appears to be rather poor. Oien[9] obtained the response of a flexible strip-footing subjected to surface waves by expanding the plate motion in terms of natural modes and then forcing the model response into compliance with the wave excitation through satisfaction of compatibility and equilibrium on the surface of the half-plane. Oien's work was extended by Iguchi[10,11] to determine the response of a rectangular plate in smooth contact with a linear half-space. Lin[12] determined analytically the harmonic vertical and rocking motion of a surface flexible disc placed on a viscoelastic half-space, while Krenk and Schmidt[13,14] considered also analytically symmetric and asymmetric vibrations of an elastic circular plate resting on an elastic half-space. Treatment of soil-structure interaction problems by the Finite Element Method (FEM) presents certain disadvantages caused by the fact that a semi-infinite medium is represented by a finite size model[15]. A very successful approach for the treatment of two-dimensional linear soil-structure interaction problems by the FEM, which is free of the disadvantages concerning the soil model, is the substructuring method of Chopra and his co-workers[3,4,16]. The effect of the flexibility of surface 2-D foundations has also been discussed by Aydinoglou and Cakiroglou[17] on the basis of Chopra's work[16].

Lately, a relatively new technique, the Boundary Element Method (BEM), has been successfully implemented for the analysis of dynamic soil-structure interaction problems involving 2-D and 3-D rigid foundations on elastic half-space[18,19]. This method is ideally suited for engineering problems involving finite domains because it reduces the spatial dimensions of the problem by one and takes into account the radiation conditions at infinity[20]. Whittaker and Christiano[21,22] and Iguchi and Luco[23] combined the BEM with the well-established Finite Element Method to study the problem of the dynamic behaviour of an elastic flexible rectangular plate placed on an elastic half-space and subjected to

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either harmonic external forces or oblique seismic waves. In all cases, with the exception of references 1, 2 and 24, a frequency domain BEM formulation has been employed to evaluate the response of foundations to dynamic disturbances. A frequency domain formulation, however, does not permit an extension to the case of nonlinear soil behaviour.

In this paper, the dynamic response of a massless flexible two-dimensional surface foundation placed on a homogeneous linear elastic half-space representing the soil medium and subjected either to external dynamic forces or obliquely incident seismic waves of a general time variation, is numerically obtained. The solution of the present boundary value problems is based on a hybrid Boundary Element-Finite Element formulation. Following the methodology presented in detail in references 2 and 25, the foundation and the soil foundation interface are discretized into a number of line elements, while the time variation of the externally applied forces is approximated by a sequence of rectangular impulses of equal duration. Then, on the assumption of constant variation of displacements and tractions over each element and time step, the FEM is used to determine the stiffness matrix of the flexible footing and the BEM is employed to determine the dynamic stiffness matrix of the soil medium. The response of the foundation to an impulse disturbance is obtained through an appropriate coupling of the compatibility and equilibrium requirements at the soil-foundation interface. Finally, the foundation response to the externally applied forces is determined by superimposing all the individual impulse responses. Solutions are presented for a flexible strip-footing subjected either to a point, uniform pressure and moment loadings or to seismic waves. For each type of dynamic loading, parametric studies are conducted to examine the effect of the relative stiffness between the plate and the foundation on the response of the plate.

The main advantage of the proposed time domain BEM–FEM formulation is that, unlike frequency domain techniques, it provides directly the transient response and forms the basis for extension to the case of nonlinear soil behaviour[1,2,25].

FORMULATION AND SOLUTION

Consider the flexible surface massless strip-footing of Fig. 1 in frictionless contact with a homogeneous isotropic linear elastic soil medium and subjected to a rectangular impulse force at time step m. Denoting the vertical displacement of the foundation by $v(x_1, t)$, the normal contact stress distribution at the soil-foundation interface by $\tau_{22}(x_1, t)$ and the magnitude of the vertical rectangular impulse force by $p(x_1, t)$, the governing equation of a plate strip-foundation can be expressed as[26]

$$D\frac{\partial^4 v(x_1, t)}{\partial x_1^4} = p(x_1, t) - \tau_{22}(x_1, t) \quad (1)$$

where $D = E_p t_p^3/12(1 - v_p^2)$ represents the flexural rigidity of the foundation plate with E_p denoting the Young's modulus, t_p the thickness, and v_p the Poisson's ratio of the plate, respectively. Under the assumption of zero initial conditions and body forces, the vertical displacement

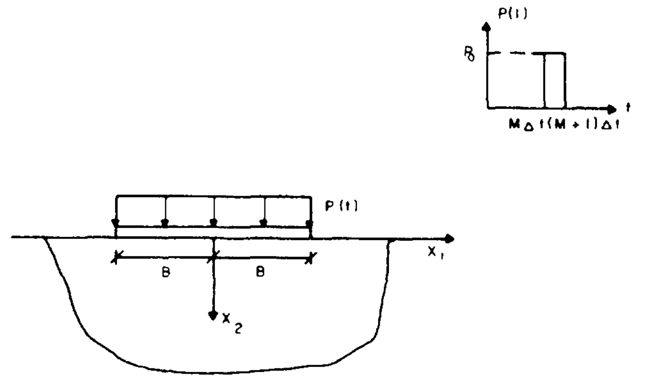

Fig. 1. Geometry of a flexible strip-foundation subjected to a rectangular impulse force

$v(x_1, t)$ at the soil-foundation interface satisfies the integral equation[2]

$$\tfrac{1}{2}v(\xi, t) = \int_S v_{22}[\xi, t; x/\tau_{22}(x, t)]\, \mathrm{d}S(x) \quad (2)$$

where S denotes the soil-foundation contact area and the component v_{22} of the Stoke's tensor for the infinite elastic space is explicitly given by[27]

$$\begin{aligned} v_{22}[\xi, t; x/\tau_{22}(x, t)] = \frac{1}{2\pi\rho}\Bigg\{ \frac{\partial^2}{\partial\xi_i\,\partial\xi_j}\Bigg[& H\left(t - \frac{r}{c_1}\right)\int_r^{c_1 t}\frac{\mathrm{d}n}{(n^2 - r^2)^{1/2}} \\ & \cdot\int_0^{t-(n/c_1)} v\tau_{22}\left(t - \frac{n}{c_1} - v\right)\mathrm{d}v \\ & - H\left(t - \frac{r}{c_2}\right)\int_r^{c_2 t}\frac{\mathrm{d}n}{(n^2 - r^2)^{1/2}} \\ & \cdot\int_0^{t-(n/c_2)} v\tau_{22}\left(t - \frac{n}{c_2} - v\right)\mathrm{d}v\Bigg] \\ & + \frac{1}{c_2^2}H\left(t - \frac{r}{c_2}\right)\int_r^{c_2 t}\tau_{22}\left(t - \frac{n}{c_2}\right) \\ & \cdot\frac{\mathrm{d}n}{(n^2 - r^2)^{1/2}}\Bigg\} \end{aligned} \quad (3)$$

$$r_i = x_i - \xi_i, \qquad r^2 = (x_i - \xi_i)(x_i - \xi_i),$$
$$n^2 = r^2 + x_3^2; \qquad (i = 1, 2) \quad (4)$$

where ρ represents the density of the soil medium, H is the Heaviside function, and c_1 and c_2 are the dilatational and shear wave velocities, respectively.

By discretizing the strip-foundation and the soil foundation interface into a Q number of elements of equal length L, the time t into n intervals of equal duration Δt and assuming a constant distribution of displacements and tractions over each element and time interval, equations (1) and (2) can be reduced to a set of linear algebraic equations. Thus, through standard finite element procedures[28], at a time step N for an impulse force acting at time $m\,\Delta t$, equation (1) can be expressed in the form

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{Bmatrix} u^N \\ \theta^N \end{Bmatrix} = \begin{Bmatrix} P_2^N \\ M_3^N \end{Bmatrix} - \begin{Bmatrix} R_2^N \\ 0 \end{Bmatrix} \quad (5)$$

where K_{ij} ($i = 1, 2$ and $j = 1, 2$) are the stiffness elements of the total stiffness matrix for the elastic strip-foundation obtained by appropriate superposition of the element stiffness matrix, $\{\mathbf{u}^N\}$ and $\{\boldsymbol{\theta}^N\}$ represent the nodal vertical displacement and rotation vectors, respectively, $\{\mathbf{P}_2^N\}$ and $\{\mathbf{M}_3^N\}$ are the nodal external forces acting on the foundation and $\{\mathbf{R}_2^N\}$ is the vector of the nodal forces associated with the contact stresses. Furthermore, the integral equation (2) can be expressed in the discretized form

$$\tfrac{1}{2}\{\bar{\mathbf{u}}^N\} = [\mathbf{DG}^M]\{\mathbf{t}^N\} + \{\mathbf{PG}^N\} \tag{6}$$

where $\{\bar{\mathbf{u}}^N\}$ and $\{\mathbf{t}^N\}$ are the vertical displacement and traction vectors, respectively, at the centres of the elements at the soil-foundation interface at time $N\,\Delta t$ due to an impulse force at time $m\,\Delta t$, while the terms of the diagonal matrix $[\mathbf{DG}^m]$ and the vector $\{\mathbf{PG}^N\}$ are explicitly given in references 2 and 25. Equation (6) solved for the traction vector yields

$$\{\mathbf{t}^N\} = [\mathbf{DG}^m]^{-1}(\tfrac{1}{2}\{\bar{\mathbf{u}}^N\} - \{\mathbf{PG}^N\}) \tag{7}$$

Consequently, the vector of the resultant forces corresponding to the contact stresses over the soil-foundation interface can be obtained from

$$\{\bar{\mathbf{R}}_2^N\} = L[\mathbf{DG}^m]^{-1}(\tfrac{1}{2}\{\bar{\mathbf{u}}^N\} - \{\mathbf{PG}^N\}) \tag{8}$$

In order to achieve compatibility between the deflection of the foundation and the motion of the ground at the interface, the average displacement over an element q is approximated by the mean value of the nodal displacements at the ends of the element q[23,25]

$$\bar{v}_q^N = \tfrac{1}{2}(v_{1q}^N + v_{2q}^N) \qquad q = 1, 2, \ldots, Q \tag{9}$$

or in matrix form for the whole foundation, by

$$\{\bar{\mathbf{u}}^N\} = [\mathbf{T}]\{\mathbf{u}^N\} \tag{10}$$

where the matrix $[\mathbf{T}]$ is of order $Q \times (Q+1)$. Similarly, the vectors of the resultant forces $\{\bar{\mathbf{R}}_2^N\}$ associated with the contact tractions $\{\mathbf{t}^N\}$ and the nodal forces $\{\mathbf{R}_2^N\}$ are related through

$$\{\mathbf{R}_2^N\} = [\mathbf{T}]^T\{\bar{\mathbf{R}}_2^N\} \tag{11}$$

where the superscript T denotes matrix transposition.

Equations (5) through (11) form a system of linear algebraic equations, which can be solved for the unknown vertical displacements $\{\mathbf{u}^N\}$ and rotations $\{\boldsymbol{\theta}^N\}$ to give

$$\{\mathbf{u}^N\} = \left([K_{11}] + \frac{L}{2}[T]^T[G^m]^{-1}[T] - [K_{12}][K_{22}]^{-1}[K_{21}]\right)^{-1} \cdot \left(\{\mathbf{P}_2^m\} + L[T]^T[\mathbf{DG}^m]^{-1}\{\mathbf{PG}^N\} - [K_{12}][K_{22}]^{-1}\{\mathbf{M}_3^m\}\right) \tag{12}$$

$$\{\boldsymbol{\theta}^N\} = [K_{22}]^{-1}(\{\mathbf{M}_3^m\} - [\mathbf{K}_{21}]\{\mathbf{u}^N\}) \tag{13}$$

It should be noted that the matrix product $(L/2)[T]^T[G^m]^{-1}[T]$ physically represents the resistance developed by the soil at the soil-foundation interface. Once the vector $\{\mathbf{u}^N\}$ of the nodal displacements is calculated, the vector $\{\bar{\mathbf{u}}^N\}$ of the contact displacements can be obtained from equation (10). Then, the tractions at the soil-foundation interface can be calculated from equation (7).

The response of the flexible strip-foundation subjected to a sequence of M rectangular impulses approximating an external load of a transient time variation can be determined by employing the principle of superposition on the individual impulse responses given by equations (12) and (13), as described in reference 2.

Consider now the same strip-foundation placed on a frictionless elastic half-space and subjected to a train of oblique seismic waves, as shown in Fig. 2. By adopting the procedure suggested by Thau[20], the total vertical displacement field $\{u_s(x_1, t)\}$ at the contact area can be decomposed into two parts, i.e.

$$\{\mathbf{u}(x_1, t)\} = \{\mathbf{u}_f(x_1, t)\} + \{\mathbf{u}_s(x_1, t)\} \tag{14}$$

where $\{\mathbf{u}_f(x_1, t)\}$ and $\{\mathbf{u}_s(x_1, t)\}$ are the vertical displacement vectors of the free and the scattered fields, respectively.

If the time variations of both the free and the scattered continuous fields are approximated by sequences of M rectangular impulses, then for a time step N equation (14) can be expressed into the following discretized form

$$\{\bar{\mathbf{u}}^N\} = \{\bar{\mathbf{u}}_f^N\} + \{\bar{\mathbf{u}}_s^N\} \qquad \text{for } N = m$$

and (15)

$$\{\bar{\mathbf{u}}^N\} = \{\bar{\mathbf{u}}_s^N\} = \{\mathbf{0}\} \qquad \text{for } N > m$$

where $\{\mathbf{u}^N\}$, $\{\bar{\mathbf{u}}_f^N\}$ and $\{\bar{\mathbf{u}}_s^N\}$ represent the total, free, and scattered vectors of the vertical displacement fields at the centres of the elements at a time step N, respectively. In essence, equation (14) expresses the fact that the response of the massless foundation at a time step N is affected only by the impulse seismic disturbance applied at that particular time step. Since the vector $\{\mathbf{PG}^N\}$ expressing the effect of all the previous time steps in equation (6) is zero, the scattered field displacements $\{\mathbf{u}_s^N\}$ can be obtained at a time step N by

$$\tfrac{1}{2}\{\bar{\mathbf{u}}_s^N\} = [\mathbf{DG}^N]\{\mathbf{t}^N\} \tag{16}$$

and the vector of the resultant forces at the contact area is related to the contact stresses through

$$\{\bar{\mathbf{R}}_2^N\} = -\frac{L}{2}[\mathbf{DG}^N]^{-1}\{\mathbf{u}_s^N\} \tag{17}$$

Furthermore, the force displacement expression corresponding to equation (5) takes the form

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} u^N \\ \theta^N \end{Bmatrix} = \begin{Bmatrix} R_2^N \\ 0 \end{Bmatrix} \tag{18}$$

Equations (16), (17) and (18) form a system of linear algebraic equations which, in view of the compatibility equations (10) and (11), can be solved for the unknown

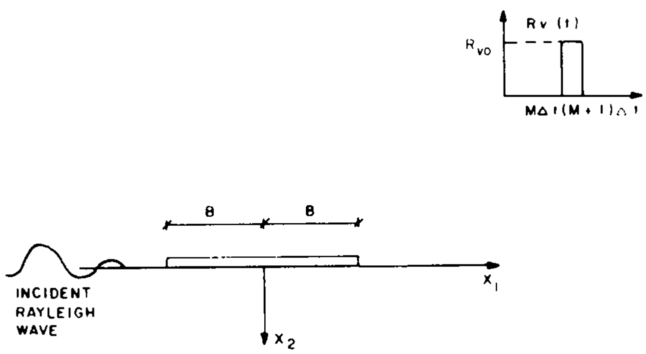

Fig. 2. Geometry of a flexible strip-foundation subjected to a rectangular impulse Rayleigh-wave excitation

nodal displacements and rotation angles of the plate to give

$$\{\mathbf{u}^s\} = [\mathbf{F}^s]\{\bar{\mathbf{u}}_f^s\}$$
$$\{\boldsymbol{\theta}^s\} = [\mathbf{K}_{22}]^{-1}[\mathbf{K}_{21}]\{\mathbf{u}^s\} \qquad (19)$$

where

$$[\mathbf{F}^s] = ([K_{11}] - [K_{11}][K_{22}]^{-1}[K_{21}] + \frac{L}{2}[T]^T[\mathbf{DG}^s]^{-1}[T]^{-1}\frac{L}{2}[T]^T[\mathbf{DG}^s]^{-1} \qquad (20)$$

Once the nodal displacements are calculated, the contact tractions can be determined from equation (7).

NUMERICAL EXAMPLE

The combined time domain BEM–FEM technique is employed to determine the dynamic response of a flexible massless strip-foundation subjected either to transient external forces or to seismic waves. The surface foundation is in smooth contact with a homogeneous isotropic linear elastic half-space characterized by a modulus of elasticity $E = 2.58384 \times 10^9$ lb/ft^2, a mass density $\rho = 10.368$ lb/sec^2/ft^4 and a Poisson's ratio $v = 1/3$ or $1/4$.

For a rigid surface massless strip-foundation, the response depends only on the elastic constants of the half-space medium and the frequency of the exciting dynamic disturbance[2]. However, the dynamic behaviour of a flexible footing is additionally affected by the material properties of the elastic plate. The main parameter characterizing the flexibility of the soil-foundation system is the relative stiffness K_r defined by

$$K_r = D_p \cdot D_s \qquad (21)$$

where the flexural rigidities D_p and D_s of the plate and soil, respectively, are given by

$$D_p = \frac{E_p t_p^3}{1 - v_p^2} \qquad (22)$$

and

$$D_s = \frac{2(1 + v_s)}{E_s B^3} \qquad (23)$$

with t_p and B representing the thickness and the half width of the foundation, respectively. Although the relative stiffness would be sufficient to characterize the elastic response of the system to a static load, it is not the only criterion to specify whether the system is stiff or flexible when a dynamic load is applied. The spatial distribution and the time variation of the dynamic disturbances are also decisive factors in determining the flexural behaviour of the foundation.

The strip-foundation under consideration has a 5 ft width and a thickness t_p that satisfies the requirement

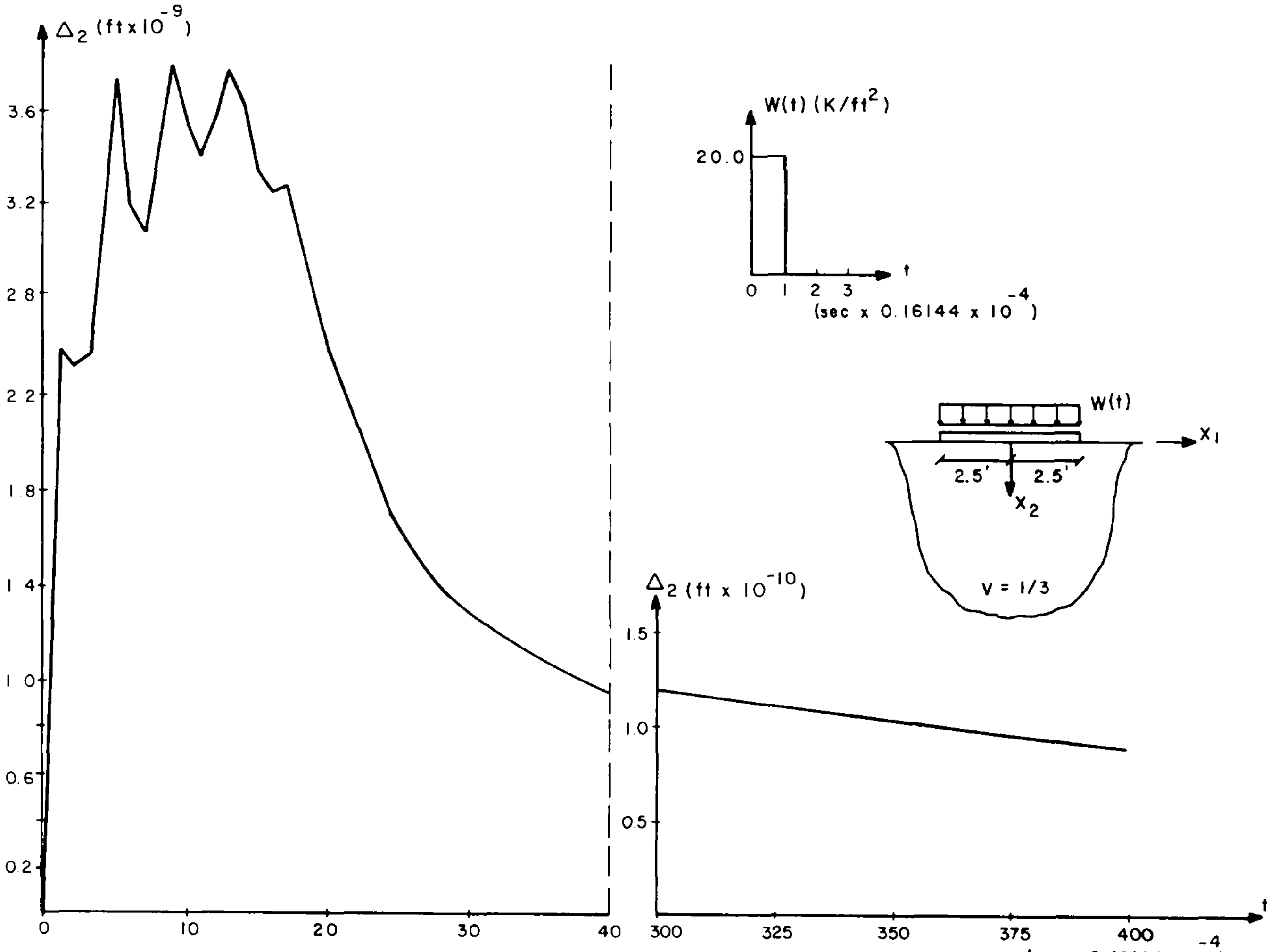

Fig. 3. Vertical uniform force impulse response at the centre

$(t_p/B) \leq 1/10$ for thin plate behaviour[26]. In all cases, the stiffness matrix of the strip-footing has been formulated for a discretization of 8 line elements of equal length L. In order to increase the accuracy of the method, the kernel matrices expressing the influence of the soil on the foundation response have been evaluated on the basis of a further discretization of the 8 elements into 3 subelements per element.

The response of the system to externally applied uniform pressure, point, and moment loadings is obtained first for a series of representative relative stiffnesses. The dynamic responses of the foundation excited in turn by a uniform impulse external loading of intensity $W_{20} = 20.0\ k/ft^2$, a point load at the centre of the plate strip of intensity $P_{20} = 100\ k/ft$ and a moment applied as a force couple of two equal, opposite point forces of intensity $P_{20} = 100\ k/ft$ located on the foundation mid-axis at distances $B/4$ from the centre line are plotted in Figs 3–8. The duration of all the impulse loads is $\Delta t = 16 \times 10^{-6}$ sec, and the relative stiffness $k_r = 0.3$. These impulse responses are sufficient to calculate the response of the soil-foundation system for any time variation of the external loadings. The vertical amplitudes of the response at the centre and one corner edge of the foundation subjected to a harmonic uniform vertical loading are plotted versus the dimensionless frequency $a_0 = B\omega/c_2$ in Figs 9 and 10. These figures have been drawn for several representative values of the relative stiffness ranging from $K_r = 0.003$ to $K_r = 3.000$. The smallest value of k_r corresponds to an almost non-existing foundation for which the response is almost identical to the free field motion, while the largest value of K_r corresponds to an almost 'rigid' plate. It is observed from Figs 9 and 10 that the displacements decrease as the frequency increases and they seem rather insensitive to changes in relative stiffness. Figure 11 portrays the amplitude of the vertical displacement along the centre line of the foundation for a representative relative stiffness and circular frequencies ω of the uniformly applied loading. Similar figures plotted for a series of relative stiffnesses can be found in reference 25. As might be expected, the displacement at the centre is greater than that at the corner edge. When the relative stiffness increases the displacement at the centre decreases, while the displacement at the corner edge either decreases or increases depending on the frequency of the applied force. The variation of the harmonic response of strip-foundations for the considered range of frequencies represents a remarkable similarity to the one reported in reference 21 for three-dimensional flexible surface foundations.

The numerical computations were performed on a Harris H100 computer. A representative impulse vertical response plotted in Fig. 3 was obtained in two steps. The first one, corresponding to the computation of the kernels for the soil, required 24.073 CP mins for 400 time steps, while the second one, which combines the calculated

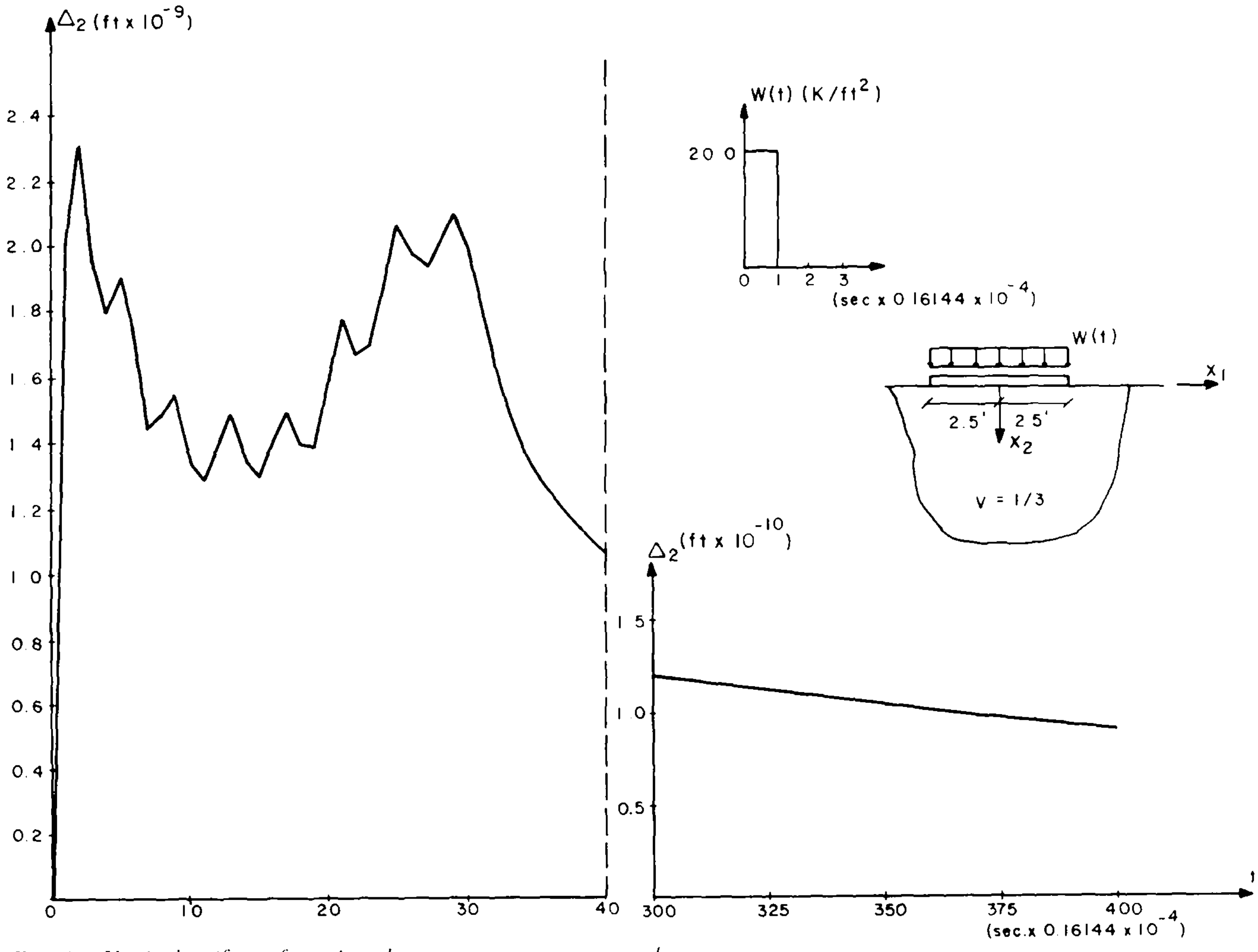

Fig. 4. Vertical uniform force impulse response at a corner edge

kernels with the stiffness matrix of the plate, required 82.00 CP secs. Only 3.85 CP secs were needed for one frequency and 1200 time steps for the harmonic vertical response.

Next, the foundation is excited in turn by a series of harmonic point loads applied along the centre line and the resulting responses are plotted versus the dimensionless frequency in Figs 12 and 13. Depending on frequency, the displacement at the centre of a 'soft' plate ($K = 0.003$) can be four to six times greater than the displacement at the centre of a 'stiff' plate ($K_r = 3.000$). The centre displacement decreases almost monotonically with increasing frequency, while for a specific value of frequency, the displacement both at the centre and the corner edge are primarily dependent on the relative stiffness. Similar behaviour has been reported in reference 21 for the centre of three-dimensional flexible surface foundations. For a specific value of relative stiffness, however, the response at the corner edge of three-dimensional plates decreases at a much faster rate than that of strip-foundations with the same width and material properties. This difference is believed to be due to the smaller elastic resistance of flexible strip plates compared with that of three-dimensional finite plates. Figure 14 corresponds to the case of concentrated harmonic forces acting at the centre and provides information analogous to that in Fig. 11. It is observed that the 'softer' the plate, the more rapidly the amplitude of the displacement decreases as the distance from the centre is increased.

Figures 15 and 16 portray the vertical displacements under one of the loads of the couple force and at the corner edge versus frequency for the case of a harmonically varying force couple at the centre of the plate and of a moment arm of $B/4$. In contrast to the two previous cases, the displacement under the load does not vary monotonically with frequency as Fig. 15 clearly shows. It actually increases for low frequencies and decreases for higher frequencies. For high frequencies, the response at the corner edge is primarily dependent on the relative stiffness. A similar behaviour has been reported in reference 21 for both the centre and the corner edge displacements of three-dimensional flexible surface foundations. Figure 17 shows the displacement amplitudes along the width of the foundation for a representative relative stiffness and series of frequencies for the case of the force couple loading. More extensive studies on the displacement variation along the width of the foundation can be found in reference 25.

In all the parametric studies presented in the above example, the foundations with small relative stiffness behaved like 'soft' plates for low frequencies and like 'stiff' plates for high frequencies. This phenomenon can be explained physically, since on the one hand a harmonic load with a low frequency produces waves of a large wavelength and, therefore, deformation that can be easily

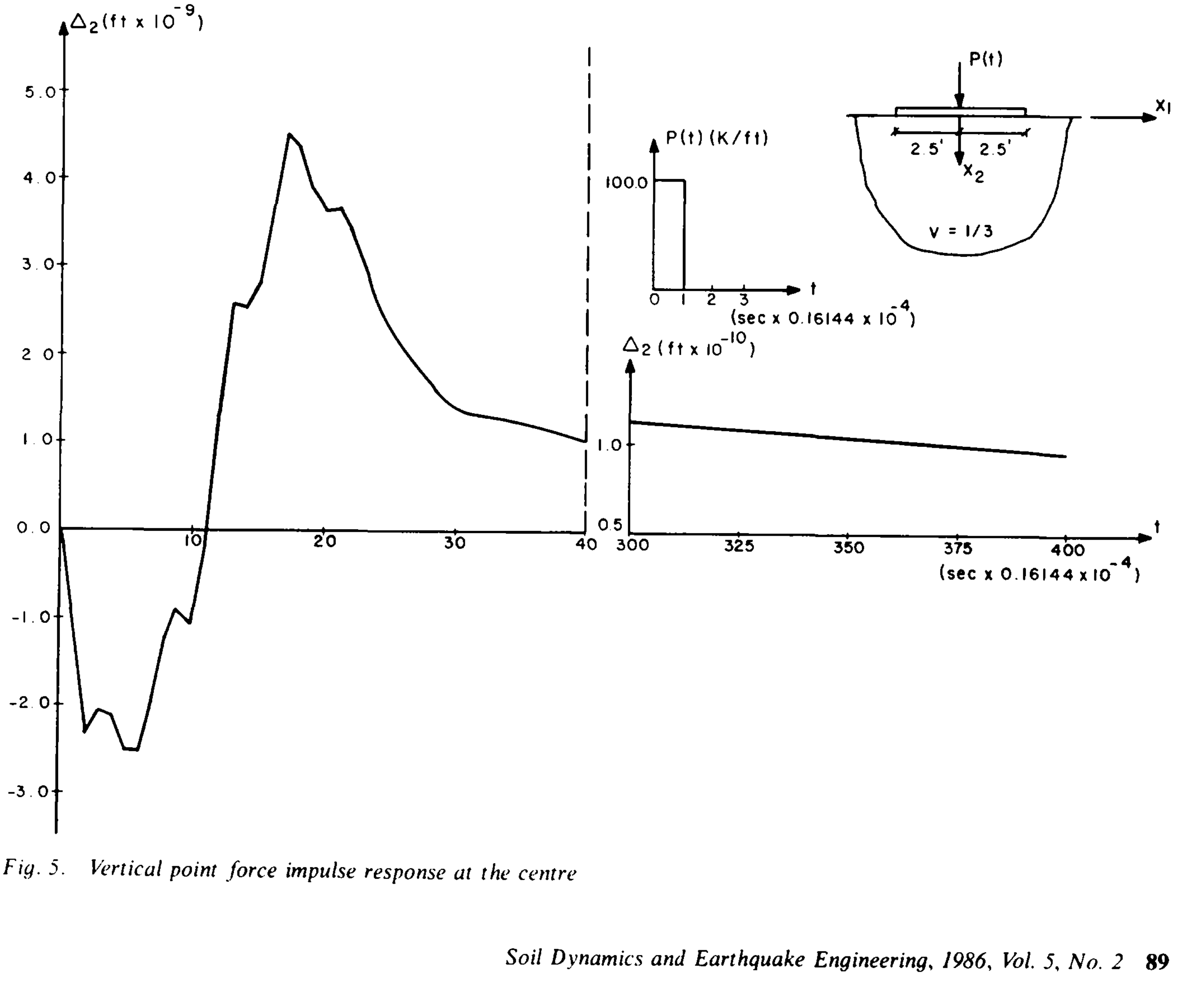

Fig. 5. Vertical point force impulse response at the centre

spanned by the plate and on the other hand the spanning of the small wavelengths caused by harmonic forces with a high frequency is difficult to achieve even by systems with small relative stiffnesses.

Consider now the same foundation subjected to a Rayleigh wave propagating in the X_1X_2 plane. It should be pointed out that the solution procedure presented herein is general and the equations derived are valid independently of the nature of the seismic dusturbance and its variation. In this example, a Rayleigh wave was so chosen as to permit comparison with the results obtained in reference 2. The calculation of the vertical response requires evaluation of the $[\mathbf{F}^N]$ given by equations (19) as well as the specification of the free-field wave motion. The matrix $[\mathbf{F}^N]$ is independent of the time variation of the seismic waves and also of the time step N under consideration, it, therefore, needs to be calculated only once for the particular soil medium and space discretization. For a elastic soil medium with a Poisson's ratio $v = 1/4$, the vertical component of the free-field surface motion u_{f2} for a harmonic Rayleigh wave horizontally propagating in the X_1X_2 plane is given by[30]

$$u_{f2} = R_v \sin\left[\omega\left(t - \frac{x_1}{9190.168}\right)\right] \tag{24}$$

Figures 18 and 19 portray the harmonic vertical response amplitude of the flexible plate to a Rayleigh wave for a sequence of frequencies and relative stiffnesses. The variation of the response amplitude along the width of the foundation is plotted in Fig. 20 for a representative relative stiffness. A more detailed study on the response variation along the width of the foundation can be found in reference 25. In Figs 18–20, it is observed that the displacement at the centre decreases with increasing frequency, while the displacement at the corner edge initially increases, but then starts to decrease at higher frequencies. Also, as expected, the response amplitude decreases with decreasing plate stiffness.

The computation of the $[\mathbf{F}^N]$ matrix for 12 elements and 5 subelements per element required 32.78 CP secs in a Harris H100 computer. For each frequency of Figs 18 and 19 only 26.63 CP secs were required. Such a considerably small amount of CP time was expected, since no superposition was employed for the solution of the seismic problem.

CONCLUSIONS

The direct time domain BEM has been combined with the FEM for the determination of the response to transient forces or wave excitations of a massless flexible strip-foundation supported at the surface of an elastic half-space.

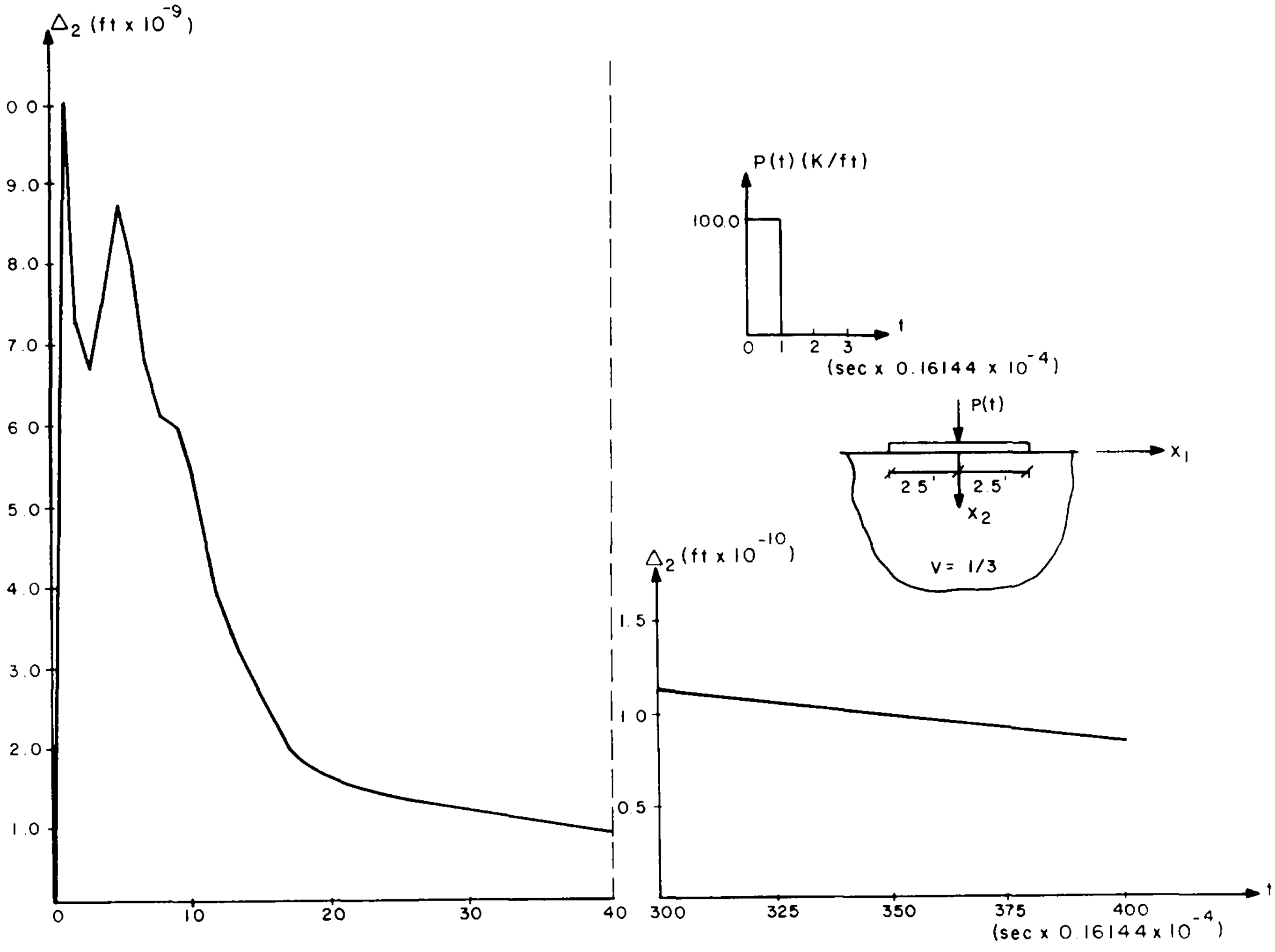

Fig. 6. Vertical point force impulse response at a corner edge

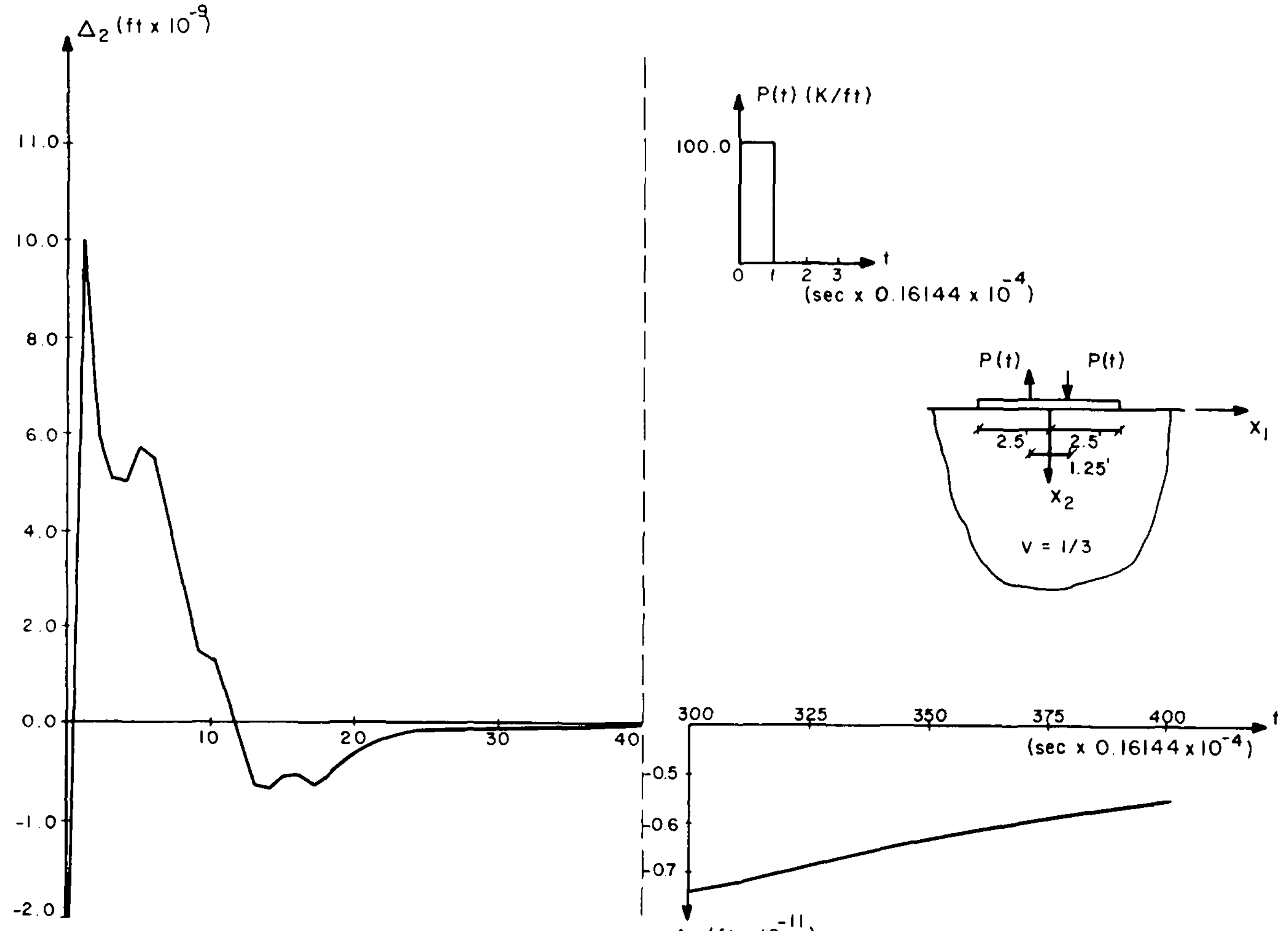

Fig. 7. Vertical impulse response under a point force

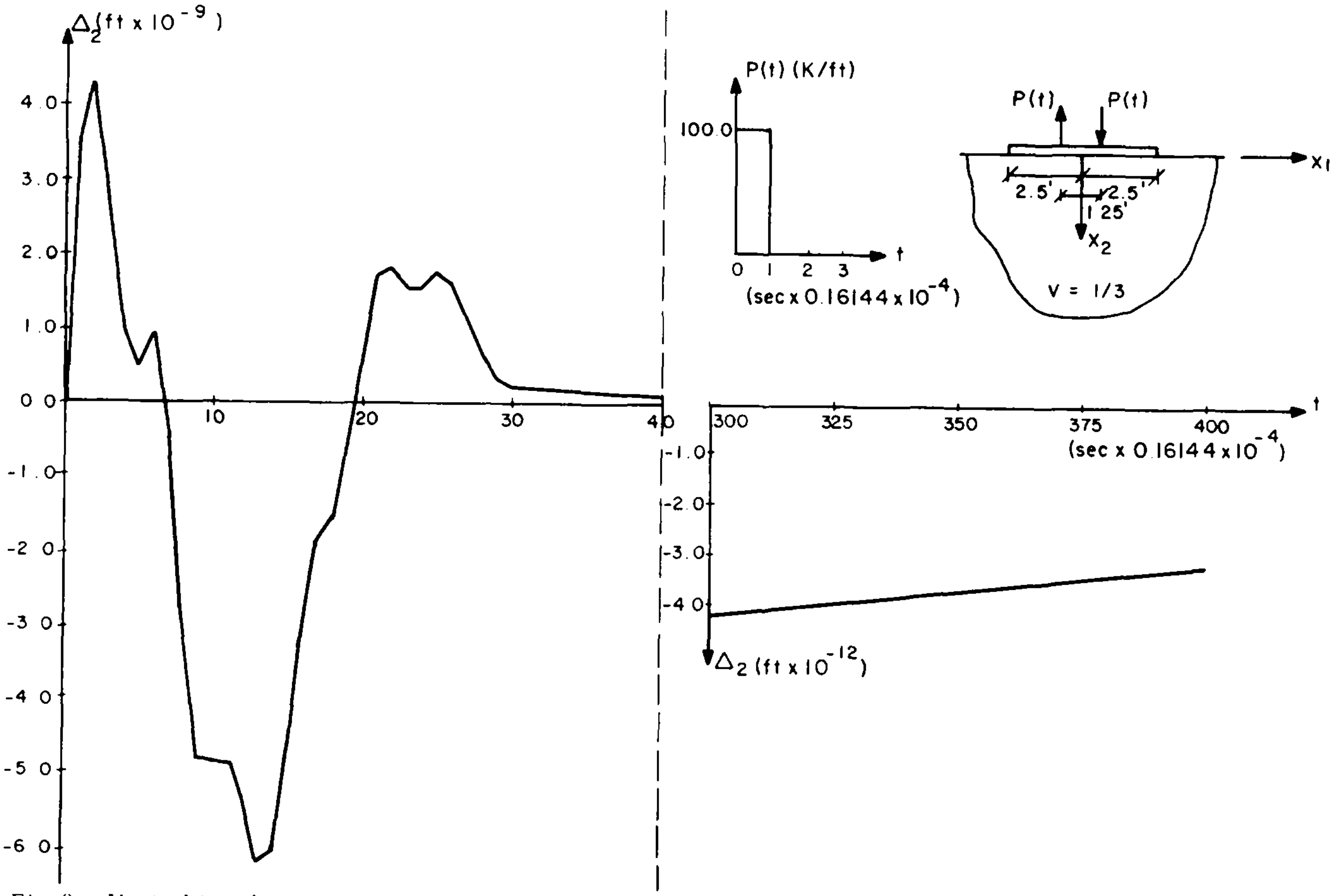

Fig. 8. Vertical impulse response at a corner edge

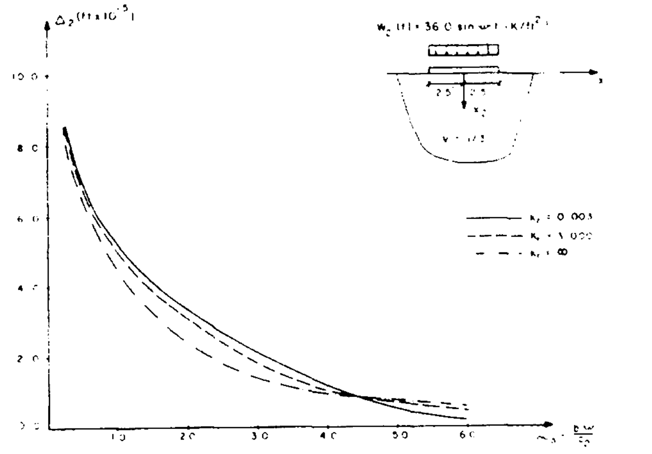

Fig. 9. Vertical harmonic uniform force response amplitude at the centre

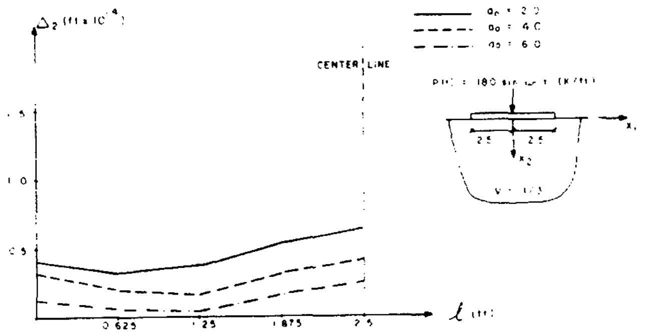

Fig. 11. Displacement profile for $K_r = 0.300$

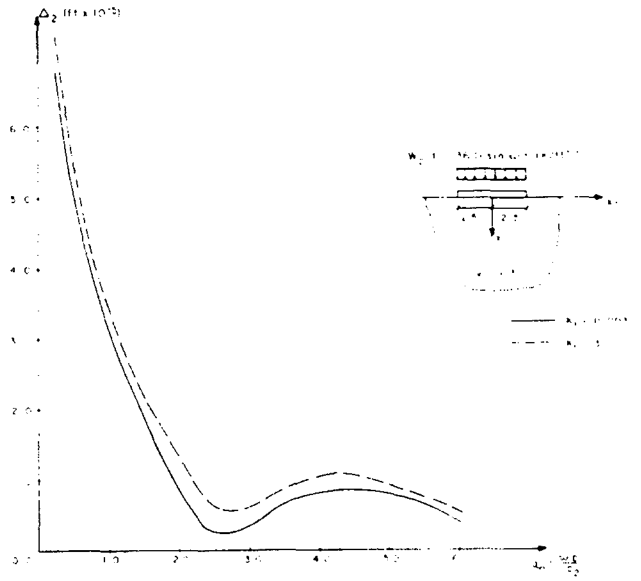

Fig. 10. Vertical harmonic uniform force response amplitude at a corner edge

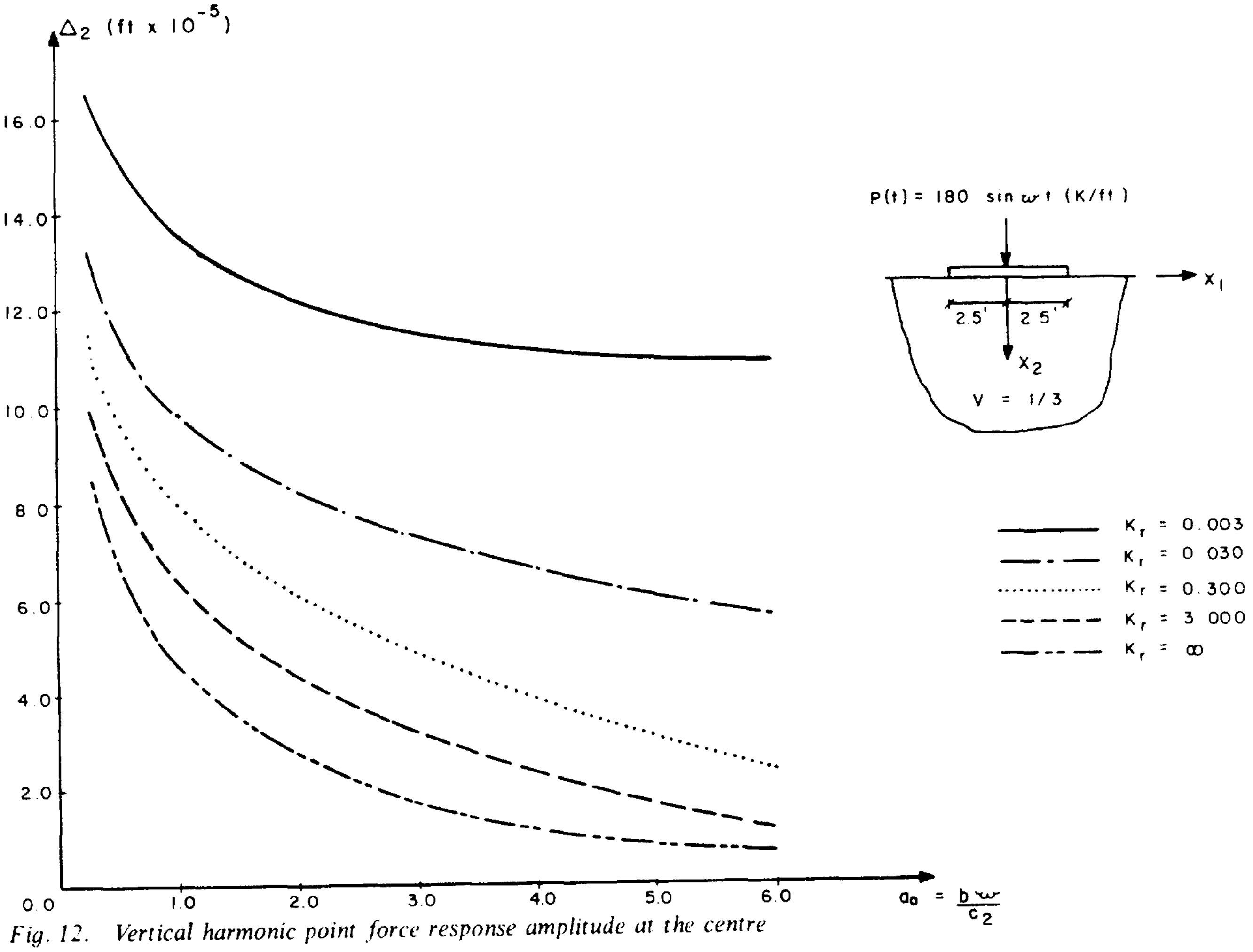

Fig. 12. Vertical harmonic point force response amplitude at the centre

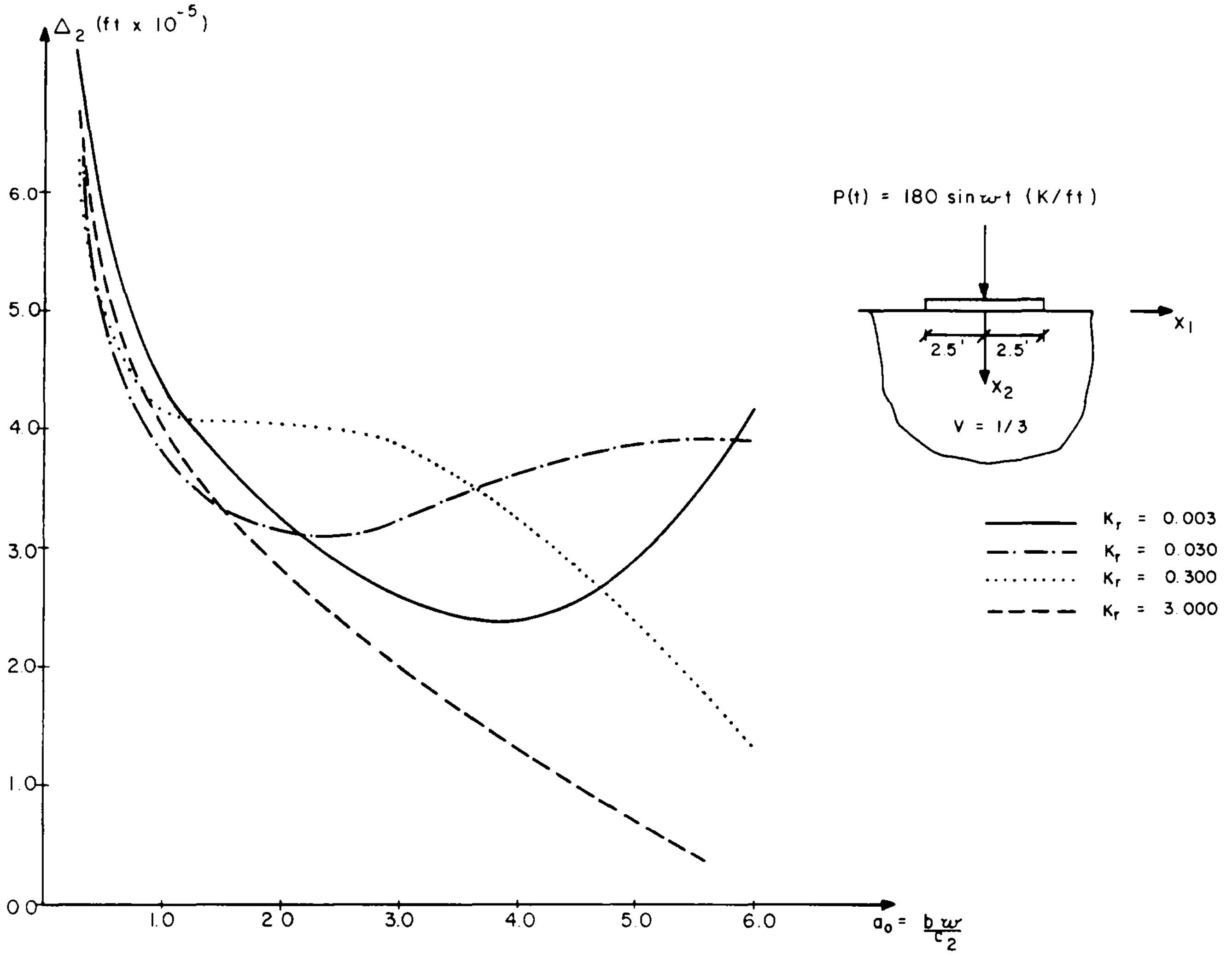

Fig. 13. Vertical harmonic point force response amplitude at a corner edge

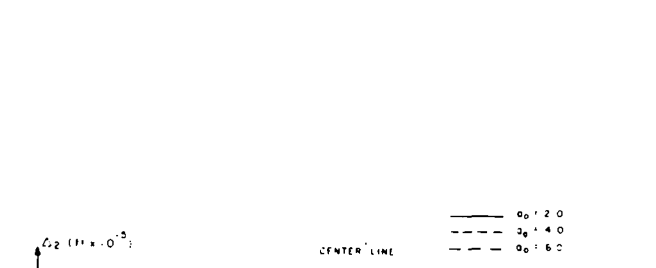

Fig. 14. Displacement profile for $K_r = 0.300$

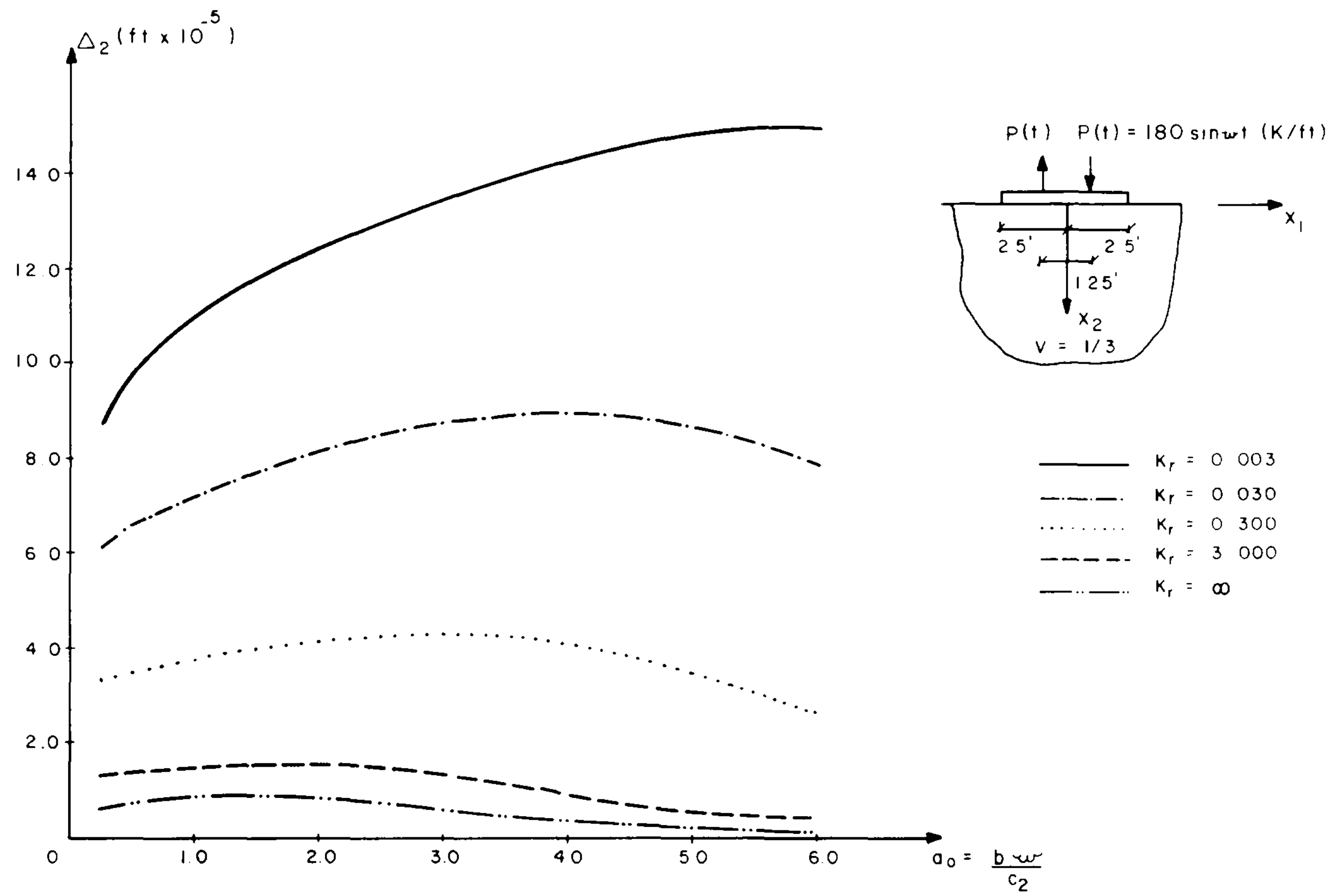

Fig. 15. Vertical harmonic response amplitude under a point load

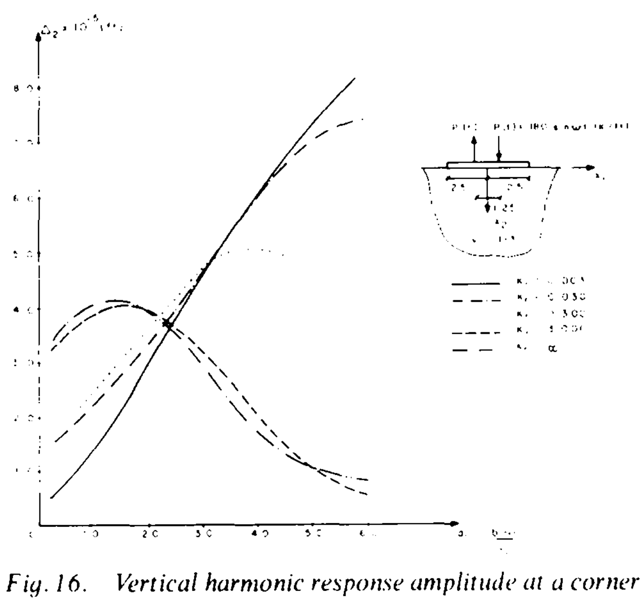

Fig. 16. Vertical harmonic response amplitude at a corner edge

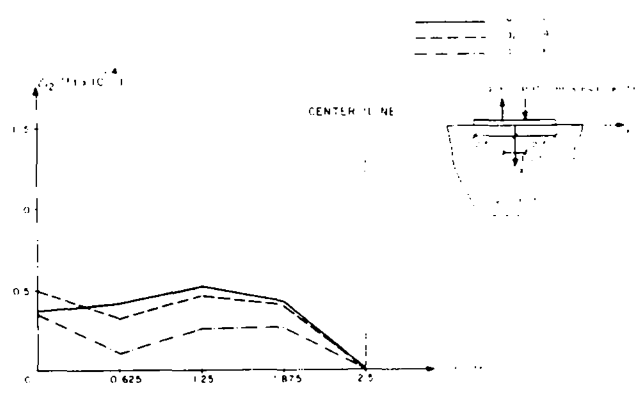

Fig. 17. Displacement profile for $K_r = 0.300$

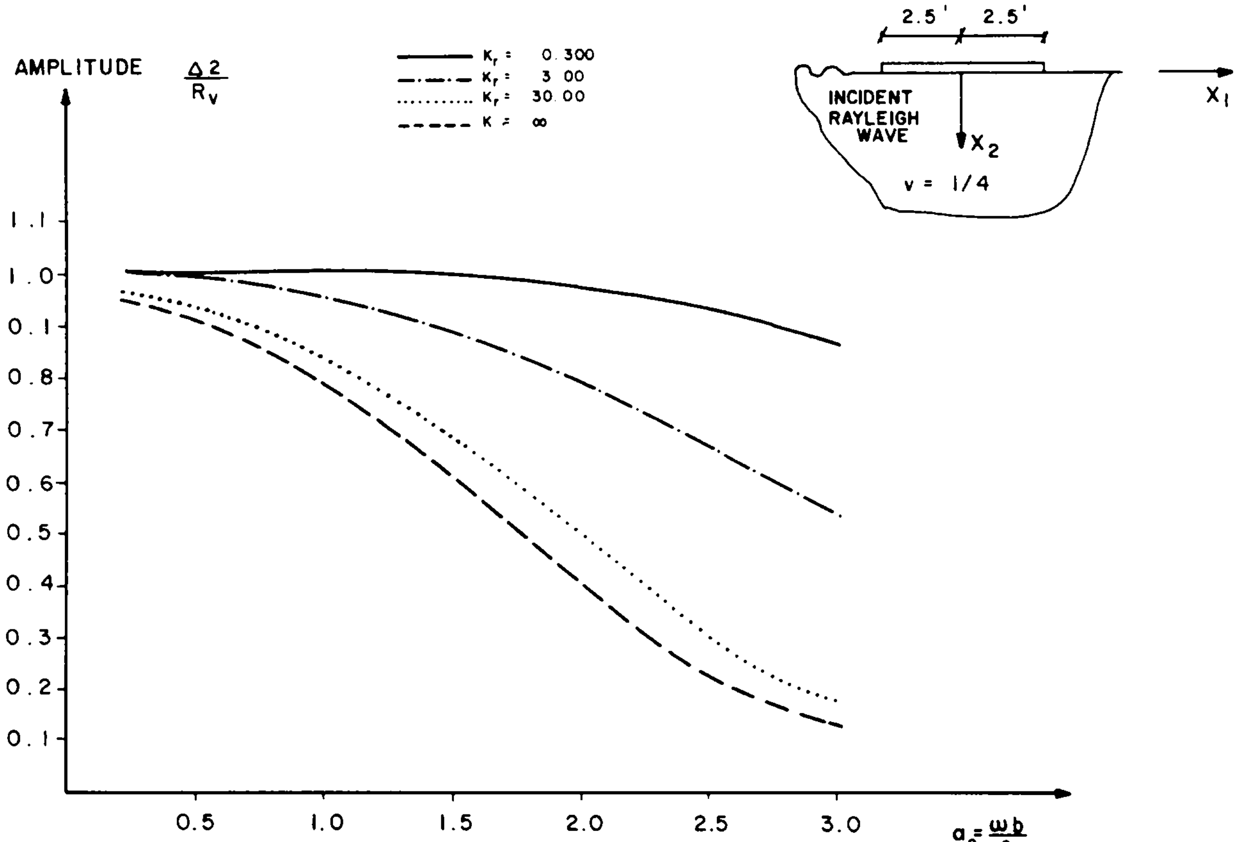

Fig. 18. Vertical harmonic Rayleigh-wave response amplitude at the centre

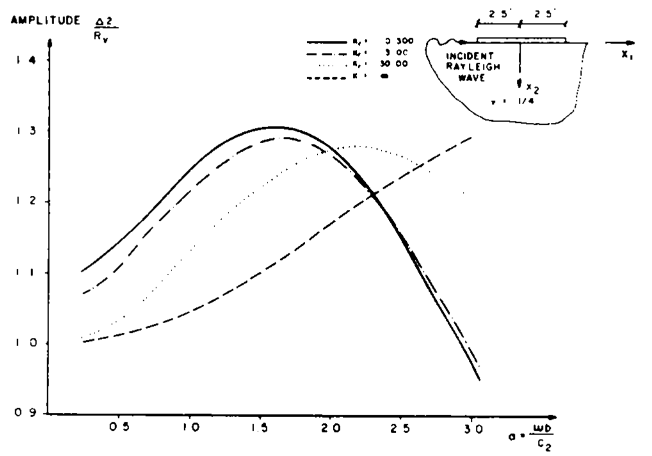

Fig. 19. Vertical harmonic Rayleigh-wave response amplitude at a corner edge

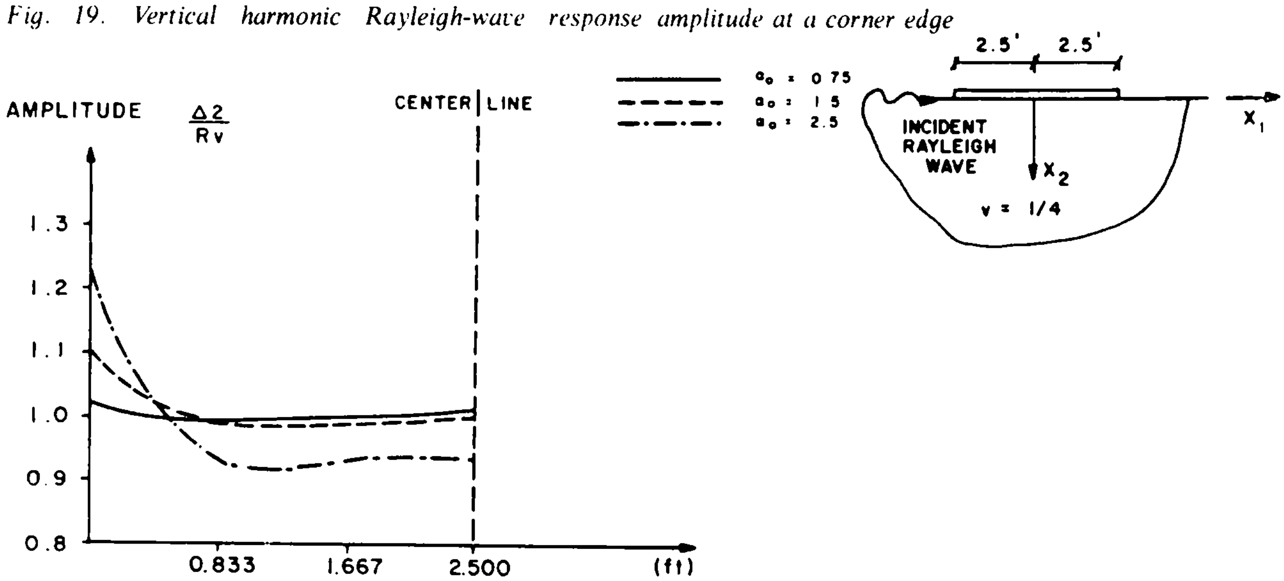

Fig. 20. Displacement profile for $K_r = 0.300$

Parametric studies are conducted for various types of external loads, relative soil-foundation stiffnesses, and frequencies of harmonic external disturbances. In all cases, the response at the centre of the footing decreases for increasing frequency of the externally applied loads. For a specific relative stiffness, however, the response at the corner edge does not vary monotonically with the frequency of the exciting dynamic disturbance. In addition to the relative stiffness and the frequency of the applied forces, the spatial distribution of the loadings is a decisive factor on the response amplitude of the foundation. In fact, for the case of an external concentrated force or moment acting at the centre of the footing, the response greatly depends on the relative stiffness, while the stiffness effect is insignificant for the case of a uniform loading. On the other hand, for the case of seismic loading and at low frequencies, the response of both flexible and stiff two-dimensional plates complies closely with the free field motion, while for higher frequencies the response amplitude greatly depends on the relative stiffness. Even though the obtained results for a strip-foundation present may similarities with the dynamic behaviour of a massless flexible surface three-dimensional finite plate, several differences due to the lack of corners have been observed.

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