Assessment of SSI on the longitudinal seismic response of short span bridges

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Current practice usually neglects the effects of soil–structure interaction (SSI) in the seismic analysis and design of bridges. This work attempts to assess the significance of SSI on the seismic response of short span bridges. The focus is placed on pier behaviour, since piers together with the abutments are the most critical elements in securing the integrity of bridge superstructures during earthquakes.

The study is based on a simple representation of a soil–bridge pier system, yet one able to capture the effects of the most significant physical parameters. It has been found that SSI greatly affects the dynamic behaviour of bridge piers leading to more flexible systems, increased damping and larger total displacements. Besides a thorough investigation of the relative significance of various physical parameters on the system response, an easy-to-use approach that can be incorporated for a preliminary design of bridges concurrent with the AASHTO specifications is presented. The study concludes that safer and more economical bridge designs can be obtained by properly accounting for SSI.

Keywords: earthquake, soil–structure interaction, bridges

Failures of bridges from earthquakes have led to the development of elaborate and realistic bridge seismic analyses and design guidelines. Extensive presentations of current commonly employed analysis and design procedures can be found in the book of Okamoto[1], the FHWA final report[2] and Bridge Design Specifications[3]. A comprehensive discussion of the literature on vibration response of highway bridges is given in a review article by Gangarao[4].

In the United States the most widely accepted design procedures are provided by the AASHTO guide specifications[5]. The AASHTO specifications recommend three methods of analysis; namely, the elastic seismic response coefficient, the single mode spectral analysis, and the multi-mode spectral analysis. In all three methods consideration is given to soil site effects, thus recognizing the importance of the local soil conditions. It should be noted, however, that the effects of soil-structure interaction (SSI) created from the presence of the bridge super- and sub-structures in the vicinity of the bridge foundations together with the foundations' influence on the structural behaviour are not considered in current practice. Simply stated, SSI is present if the soil-foundation interface moves or distorts differently from the corresponding soil surface of the free field. The limited number of bridge studies considering SSI can be primarily attributed to the complexity of the physical problem and the lack of an easy-to-use design approach that can account for SSI[6–8]. Further, recent experimental and analytical studies have identified the significant role that SSI can play during seismic excitations of bridges, and have demonstrated the need to incorporate SSI in the design of a wide class of bridge structures[6,9,10].

The primary objective of this study is twofold: first, to assess the effect of soil-structure interaction on the longitudinal seismic response of bridges and, second, to develop an approach that can account for SSI and can be easily incorporated in a preliminary design of bridge piers.

Bridge-soil system and method of analysis

Consider the bridge-soil system shown in *Figure 1* that has a deck considerably stiffer than the piers and is excited by a seismic ground motion acting along the longitudinal direction. The bridge span lengths are short, and the piers are assumed to be identical in size and material properties. Further, the mass of the piers is

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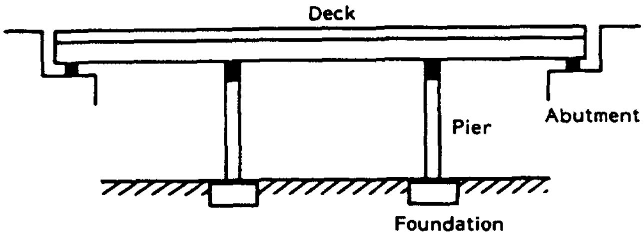

Figure 1 Typical elevation of short span highway bridge

considerably smaller than the mass of the bridge deck. Under these assumptions, the longitudinal dynamic response of the bridge can be simulated with aid of the three-degrees-of-freedom model shown in *Figure 2*. The three degrees of freedom include the total lateral displacement of the bridge deck, u_t, the horizontal displacement of the foundation relative to the free-field motion, u_o, and the rotation of the system at the foundation level, θ. In the bridge model, the piers and the foundations are assumed to be massless. In order to simplify the analysis, all piers are identical in size and stiffness. Consequently the tributary mass for each pier, m, has been obtained by dividing the overall mass of the bridge deck by the number of piers. The height and the flexural stiffness of the pier are denoted as h and k, respectively. The overall damping in the pier is hysteretic and is characterized by a damping ratio, ζ. It should be noted that the assumption of a rigid deck greatly simplifies the dynamic analysis of the system by restricting the rotational degrees of freedom at the top of the piers with very little sacrifice in computational accuracy[11].

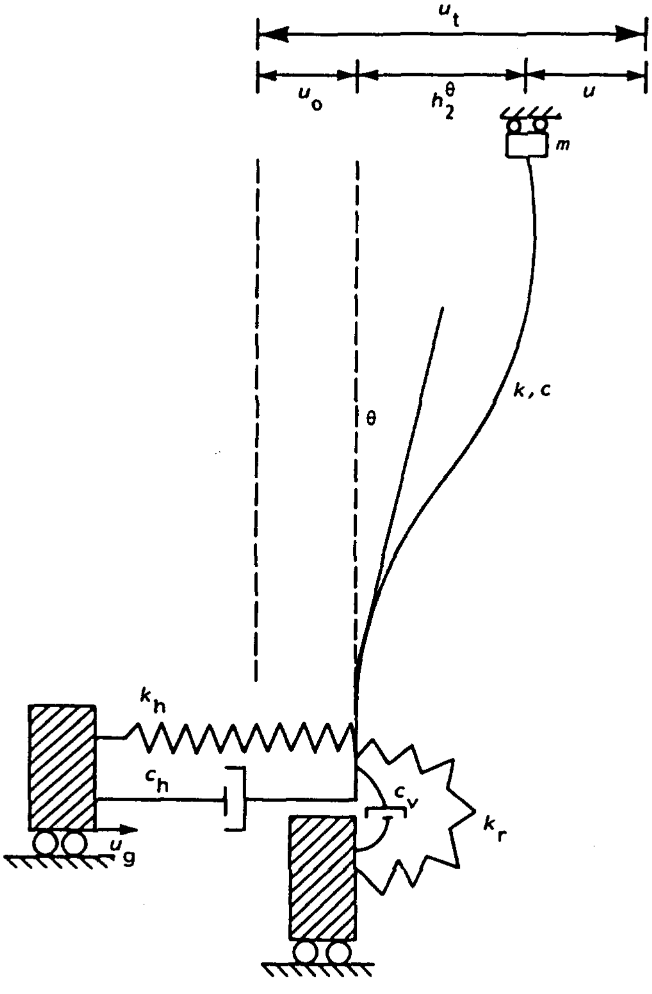

Figure 2 Model of built-in pier–soil system to horizontal excitation

In the bridge model, the soil supporting the piers through a massless foundation is modelled as spring dampers acting in the horizontal and rotational directions. Viscous damping is used to simulate the radiation damping in the soil, which is developed through the loss of energy emanating from the foundation in the semi-infinite soil medium. The material damping occurring in the soil is hysteretic and is characterized by a damping ratio ζ_g. The nomenclature used for the foundation damping and stiffness properties is indicated in *Figure 2*. Making use of the correspondence principle[12], the amplitudes of the horizontal force, P_h, and moment, M_r, that develop at the base for a harmonic ground motion $u_g e^{i\omega t}$ can be written in the following form

$$P_h = k_h(1 + 2\zeta_h + 2\zeta_g i)u_o \quad (1)$$

and

$$M_r = k_r(1 + 2\zeta_r + 2\zeta_g i)\theta$$

where the horizontal, ζ_h, and rotational, ζ_r, damping ratios are given by

$$\zeta_h = \frac{\omega c_h}{2k_h} \quad \text{and} \quad \zeta_r = \frac{\omega c_r}{2k_r} \quad (2)$$

Considering the equilibrium of the horizontal forces and moments at the base of the pier and the horizontal forces acting on the mass, m, leads to the equations of motion of the bridge-soil system

$$\begin{bmatrix} \frac{\omega_s^2}{\omega^2}(1 + 2\zeta i) - 1 & -1 & 0.5 \\ -1 & \frac{\omega_h^2}{\omega^2}(1 - 2\zeta_h i + 2\zeta_g) - 1 & 0.5 \\ -1 & -1 & \begin{pmatrix} \frac{2\omega_r^2}{\omega^2}(1 + 2\zeta_r i + 2\zeta_g i) \\ + \frac{1}{6}\frac{\omega_s^2}{\omega^2}(1 + 2\zeta i) \end{pmatrix} \end{bmatrix} \begin{Bmatrix} u \\ u_o \\ h\theta \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} u_g \quad (3)$$

where the subscribed parameters ω_s, ω_h and ω_r pertain to the fixed base pier and are expressed by

$$\omega_s^2 = \frac{12EI}{h^3 m}, \omega_h^2 = \frac{k_h}{m} \quad \text{and} \quad \omega_r = \frac{k_r}{mh^2}$$

It is well established that the stiffness and damping characteristics of soil depend on the frequency content of the externally applied loads[13,14]. Nevertheless, the following frequency-independent coefficients can be used to obtain a sufficiently accurate design for a rigid, circular surface foundation with a radius a on deep soil strata[6,14]

$$k_h = \frac{8Ga}{2-\gamma}, \quad C_h = \frac{4.6Ga^2}{(2-\gamma)C_s}$$
$$k_r = \frac{8Ga^3}{3(1-\gamma)}, \quad C_r = \frac{0.4Ga^4}{(1-\gamma)C_s} \tag{4}$$

It should be noted that when the foundation is embedded or a pile foundation is used to support the piers, additional stiffness is provided to the system. Such an additional stiffness may enhance or reduce the effects of SSI, depending on the relative stiffness between the foundation and soil[15,16]. Although the bridge-soil system portrayed in *Figure 2* has three degrees of freedom, only one of them is dynamic, since all inertia quantities except the horizontal lumped mass along the longitudinal direction of the bridge are neglected. Consequently, an equivalent single-degree-of-freedom system can be derived from the three-degrees-of-freedom model via dynamic equilibrium. The equation of motion of the equivalent single-degree-of-freedom (SDOF) system subjected to a harmonic ground excitation $u_g e^{i\omega t}$ is given by

$$\left(1 + 2\bar{\zeta}i - \frac{\omega^2}{\bar{\omega}^2}\right)u = \frac{\omega^2}{\bar{\omega}^2}\bar{u}_g \tag{5}$$

in which $\bar{\omega}^2$ and $\bar{\zeta}$ are given by

$$\bar{\omega}^2 = \frac{1}{1/\omega_s^2 + 1/\omega_h^2 + (12\omega_r^2 + \omega_s^2)/3} \tag{6a}$$

$$\bar{\zeta} = \left(\frac{\bar{\omega}^2}{\omega_s^2} + \frac{3\omega_s^2\bar{\omega}^2}{(12\omega_r^2 + \omega_s^2)^2}\right)\zeta + \left(1 - \frac{\bar{\omega}^2}{\omega_s^2} - \frac{3\omega_s^2\bar{\omega}^2}{(12\omega_r^2 + \omega_s^2)^2}\right)\zeta_g + \frac{\bar{\omega}^2}{\omega_h^2}\zeta_h + \frac{36\bar{\omega}^2\omega_r^2}{(12\omega_r^2 + \omega_s^2)^2}\zeta_r \tag{6b}$$

and

$$\bar{u}_g = \frac{\bar{\omega}^2}{\omega_s^2}u_g \tag{6c}$$

The equivalent SDOF system has been derived under the assumption of maintaining the same mass, m, with the three-degrees-of-freedom system governed by equation (3), and by enforcing equal relative displacement amplitudes, u, at resonance in both systems. The equivalent damping ratio, $\bar{\zeta}$, has been evaluated at resonance, i.e., $\bar{\omega} = \omega$, and then used for the whole frequency range. It is worth noting, as can be observed from equations (6a) and (6c), that the amplitude of the exciting ground motion of the equivalent system, $\bar{u}_g$, is always smaller than the ground displacement amplitude of the three-degrees-of-freedom pier-soil model, u_g^I. The approach employed to arrive at equations (6) can lead to essentially the same relative displacements, u, in both the three-degrees and the single-degree-of-freedom systems[12]. It should also be noted that besides the simplicity of the expressions for $\bar{\omega}$ and $\bar{\zeta}$, this approach does not require use of assumed modes that satisfy the geometric boundary conditions at the ends of the pier[5,13]. The succinct expressions of the equivalent system facilitate the understanding of the influence that SSI has on the seismic behaviour of bridge structures.

Assessment of SSI

Under a seismic excitation, the interdependence of the bridge super-/sub-structure and soil can be better understood by studying the variation of the system dynamic properties expressed in terms of the dimensionless parameters

$$\bar{p} = \frac{12EI}{Gah^2}, \quad \bar{h} = \frac{h}{a} \quad \text{and} \quad \bar{m} = \frac{m}{\rho a^3} \tag{7}$$

With the aid of the nondimensional parameters, equations (6) can be cast into the following form

$$\frac{\bar{T}}{T} = \left[1 + \frac{\bar{p}}{8}\left((2-\gamma) + \frac{24(1-\gamma)\bar{h}^2}{32 + (1-\gamma)\bar{p}\bar{h}^2}\right)\right]^{1/2} \tag{8a}$$

$$\bar{\zeta} = D_{sb} + D_{mb} + D_{rb} \tag{8b}$$

in which

$$D_{sb} = \left(\frac{T}{\bar{T}}\right)^2 \times \left(1 + \frac{3(1-\gamma)^2\bar{p}^2\bar{h}^4}{1024 + 64(1-\gamma)\bar{p}\bar{h}^2 + (1-\gamma)^2\bar{p}^2\bar{h}^4}\right)\zeta$$

$$D_{mb} = 1 - \left(\frac{T}{\bar{T}}\right)^2 \times \left(1 + \frac{3(1-\gamma)^2\bar{p}^2\bar{h}^4}{1024 + 64(1-\gamma)\bar{p}\bar{h}^2 + (1-\gamma)^2\bar{p}^2\bar{h}^4}\right)\zeta_g \tag{9}$$

and

$$D_{rb} = \left(\frac{T}{\bar{T}}\right)^3 \sqrt{\frac{\bar{p}^3}{\bar{m}}} \times \left[\frac{4.6}{128}(2-\gamma) + \frac{7.2(1-\gamma)\bar{h}^2}{1024 + 64(1-\gamma)\bar{p}\bar{h}^2 + (1-\gamma)^2\bar{p}^2\bar{h}^4}\right]$$

where $\bar{T}$ denotes the period of the equivalent SDOF system including SSI, and T is the period of the fixed base structure. Equation (8b), expressing the equivalent damping $\bar{\zeta}$, consists of three components which pertain to structural material damping, D_{sb}, soil material damping, D_{mb}, and soil radiation damping, D_{rb}.

Figure 3 shows the variation of $\bar{T}/T$ as a function of $\bar{p}$ for representative values of $\bar{h}$. It should be noted that for most practical situations[6] the range that $\bar{p}$ could vary is between the values 3 and 8. For the evaluations shown in *Figure 3* and all the subsequent figures, a Poisson's ratio of $\gamma = 0.4$ has been used for the soil. Two predominant trends can be observed. First, decreasing the soil stiffness results in increasing $\bar{T}/T$ and second, increasing $\bar{h}$ also leads to larger values of $\bar{T}/T$. It should be noted that in the latter case a dramatic increase of $\bar{T}/T$ is observed even for very small values of $\bar{p}$. Since the variation of the ratio $\bar{T}/T$ characterizes the effect of soil-structure interaction, it can be deduced that SSI should be considered in

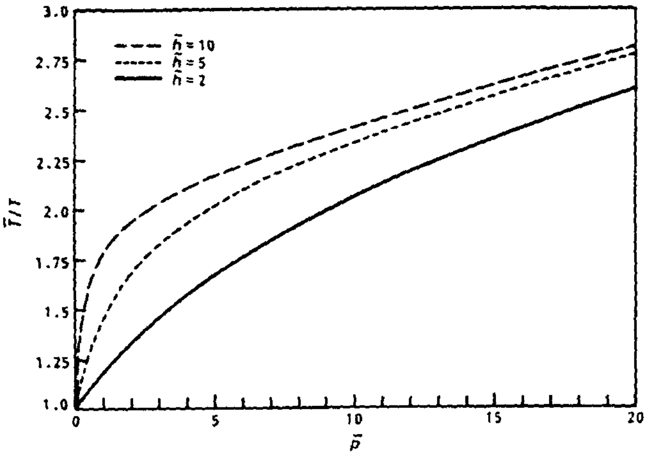

Figure 3 Natural period ratio of pier to horizontal excitation

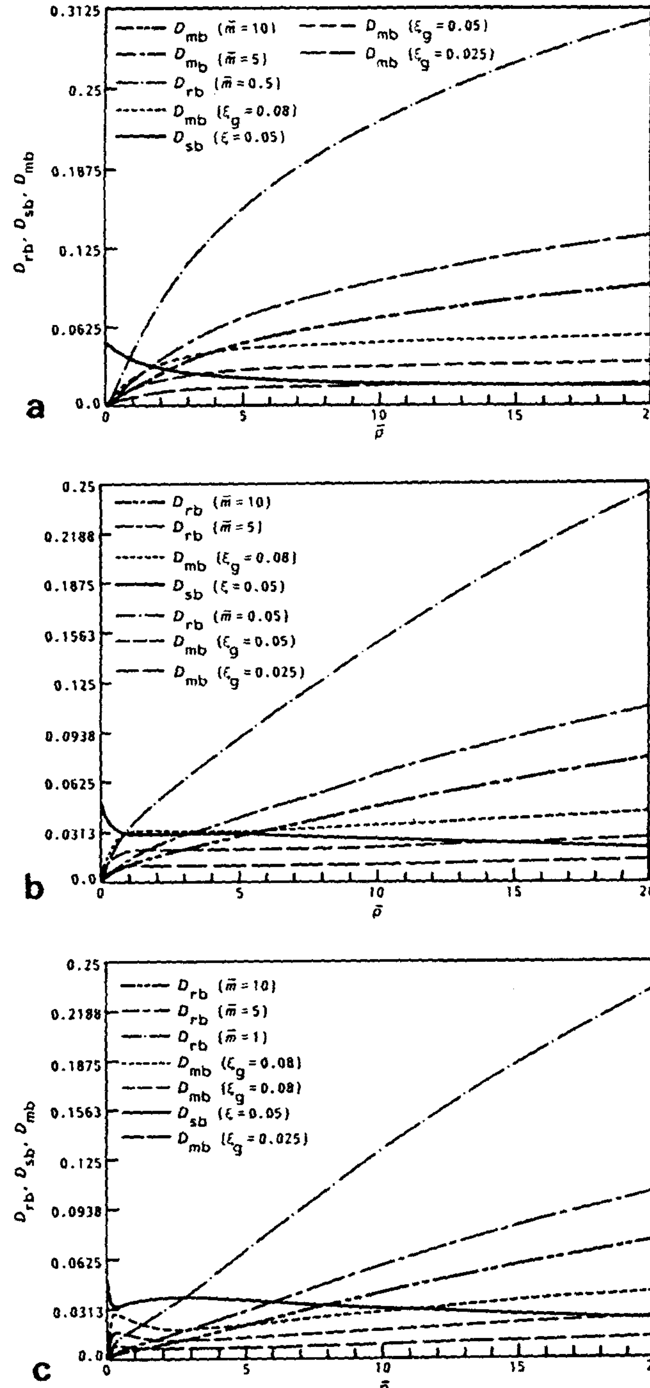

Figure 4 Variation of damping components for: (a) $\bar{h}=2$; (b) $\bar{h}=5$; (c) $\bar{h}=10$

the design of stiff bridges on flexible soil. *Figures 4a, b* and *c* depict the variation of the equivalent damping components for three representative values of $\bar{h}$. It is observed that the structural material damping contribution D_{sb} is present even for high values of $\bar{p}$, and it is more significant in slender piers than in squat piers. On the contrary, the soil material damping contribution, D_{mb}, does not approach the total soil material damping ζ_g even for soft soil conditions. The curves that pertain to soil radiation damping, designated with D_{rb} in *Figure 4*, indicate that radiation damping is higher for small values of mass ratio $\bar{m}$. Hence, higher radiation damping can be obtained by increasing the soil density through compaction, by decreasing the structural height and finally by increasing the dimensions of the foundation while maintaining the pier height unchanged. The three components of equivalent damping are added and plotted as a function of $\bar{p}$ in *Figures 5a, b* and *c* for the same representative values of $\bar{h}$ used in *Figure 4*. The structural material damping ratio, ζ, and the soil material damping ratio, ζ_g, are assumed to be 0.05 and 0.08, respectively. The selected value of ζ characterizes reinforced concrete piers, while the value of ζ_g represents a realistic value of hysteretic soil damping during strong ground motions. It should be noted that in *Figure 5* only one of the curves corresponding to $D_{rb}=0$ does not include radiation damping. If radiation damping is considered, the equivalent damping, $\bar{\zeta}$, is greater in squat piers than in slender piers. This should be mostly attributed to the presence of radiation damping which, as mentioned earlier, is more significant for squat piers. It is worth noting that, as indicated by equations (9), in absence of radiation damping and for the soil material damping ratio ζ_g being smaller than the structural damping ratio ζ, the equivalent damping $\bar{\zeta}$ will be smaller than the fixed base structural damping ζ.

Evaluation of the base shear

In order to better realize the ramifications that accounting for SSI has on the seismic behaviour of bridge piers, certain critical design quantities, such as the shear at the base of a pier, need to be evaluated in terms of the dimensionless parameters. When the effects of SSI are neglected, the shear at the base of a pier may be determined as recommended by the current Guide Specification of Seismic Design of Highway Bridges (1983)

$$V = C_s W \tag{10}$$

where C_s is the seismic design coefficient and W denotes the gravity weight associated with one pier. The seismic design coefficient can be obtained from the following expression

$$C_s(T, \zeta) = 1.2\,\frac{AS}{T^{2/3}} \tag{11}$$

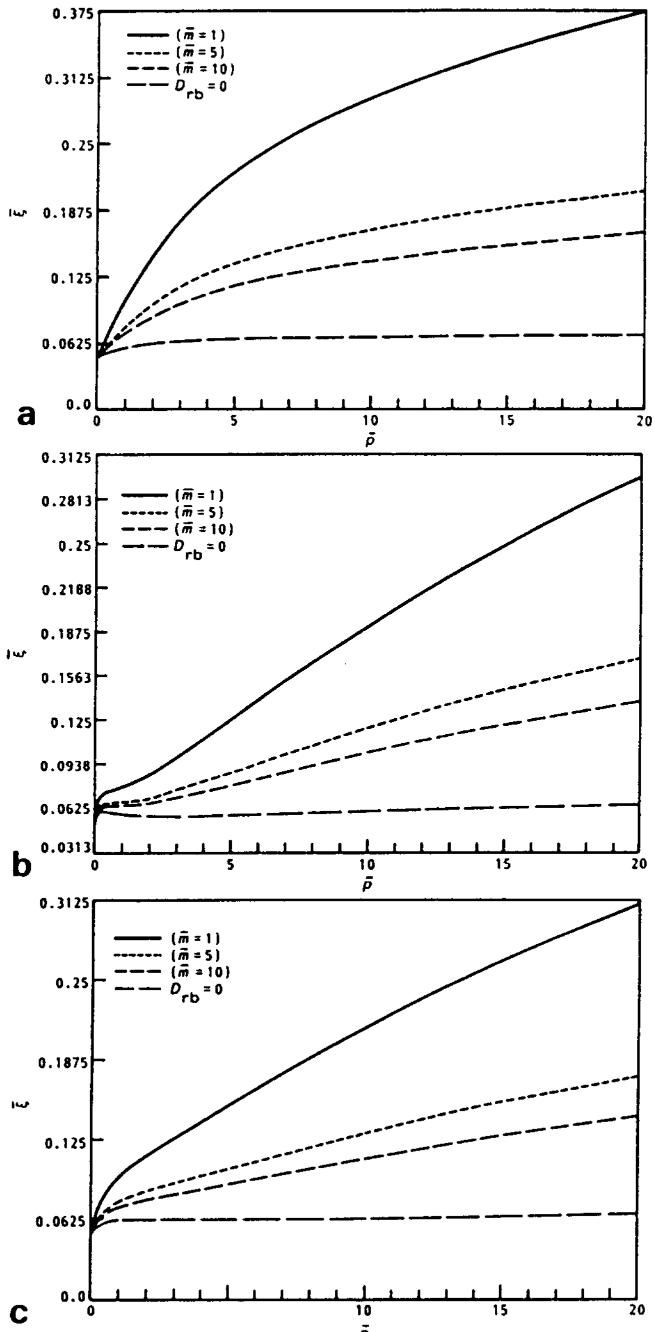

Figure 5 Equivalent damping ratio for horizontal excitation and: (a) $\bar{h} = 2$; (b) $\bar{h} = 5$; (c) $\bar{h} = 10$

From equation (11) the value of C_s corresponding to a soil-structure system with natural period $\bar{T}$, and damping of the fixed base structure, ζ, can be evaluated from

$$C_s(\bar{T}, \zeta) = 1.2\frac{AS}{\bar{T}^{2/3}} \tag{12}$$

For the most commonly encountered soil conditions, the values of C_s corresponding to different damping ratios, $\bar{\zeta}$, but to the same natural period, $\bar{T}$, can be approximately determined from[18]

$$C_s(\bar{T}, \bar{\zeta}) = C_s(\bar{T}, \zeta)\left(\frac{\zeta}{\bar{\zeta}}\right)^{0.4} \tag{13}$$

In view of equation (10), the following expression that accounts for SSI can be contemplated to provide the base shear for the pier-soil system

$$\bar{V} = C_s(\bar{T}, \bar{\zeta})W \tag{14}$$

It should be pointed out that $C_s(\bar{T}, \bar{\zeta})$ should be evaluated for the natural period and damping of the equivalent SDOF system with functional period $\bar{T}$ and damping ratio $\bar{\zeta}$. Combining equations (12), (13) and (14), the ratio of the base shear accounting for SSI, $\bar{V}$, to the fixed base shear, V, can be expressed as

$$\frac{\bar{V}}{V} = \frac{C_s(\bar{T}, \bar{\zeta})}{C_s(T, \zeta)} = \left(\frac{T}{\bar{T}}\right)^{2/3}\left(\frac{\zeta}{\bar{\zeta}}\right)^{0.4} \tag{15}$$

In this final form, the ratio $\bar{V}/V$ can also be expressed as a function of the dimensionless parameters $\bar{p}$, $\bar{h}$ and $\bar{m}$. *Figures 6a, b* and *c* present the variation of $\bar{V}/V$ as a

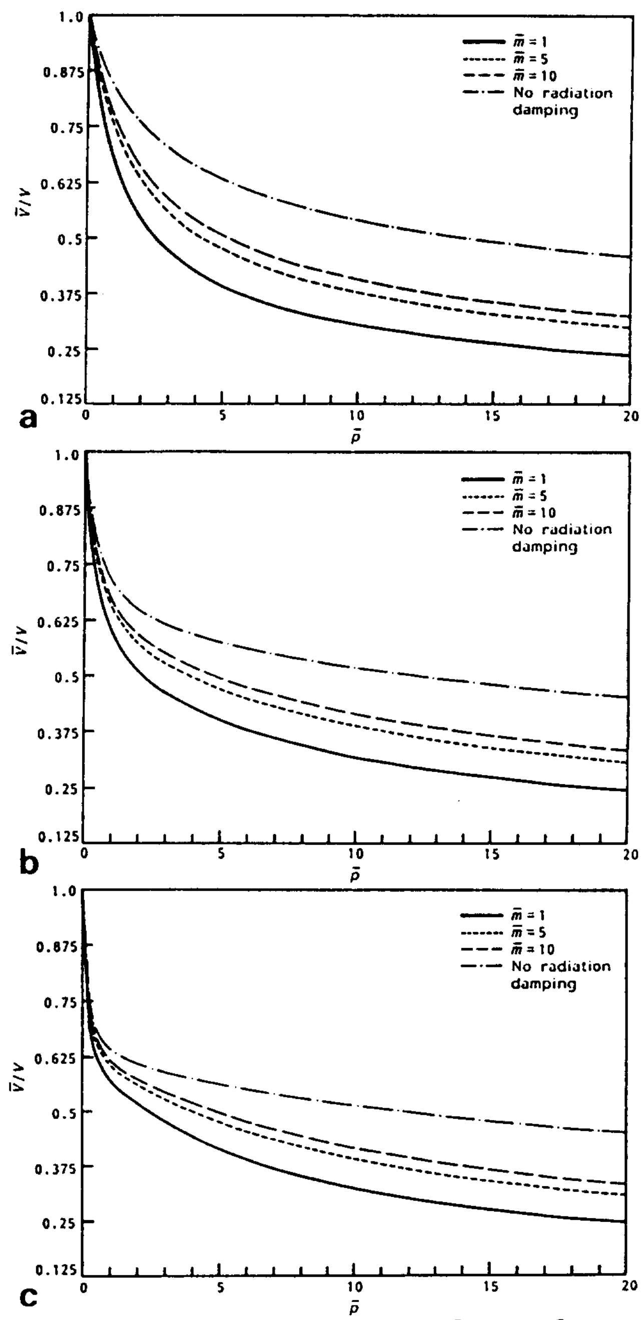

Figure 6 Shear force reduction factor for: (a) $\bar{h} = 2$; (b) $\bar{h} = 5$; (c) $\bar{h} = 10$

function of $\bar{p}$ for representative values of $\bar{m}$ and $\bar{h}$. The structural material damping ratio, ζ, and the soil material damping ratio, ζ_g, are assumed to be 0.05 and 0.08, respectively. As expected, the pier base shear decreases for decreasing soil stiffness and/or increasing the foundation-soil contact area. Attention should be paid in the design, however, that a decrease in soil stiffness could lead to an increase of the total drift at the top of the pier relative to the base, which in turn would increase the secondary shear associated with P-δ effects. Such an increase is generally small and is usually neglected in the design[1]. Three predominant trends can be observed from studying *Figure 6*. First, the shear reduction presents a sharp decrease for small values of $\bar{p}$. This behaviour is of major importance in the design of bridge piers, since the majority of short span bridges correspond to small values of $\bar{p}$. Second, the effects of mass ratio m on $\bar{V}/V$ is more apparent in squat structures. This can be attributed to the significant influence that radiation damping has on the response of bridges with short piers. The significance of radiation damping for the whole range of $\bar{p}$ is clearly demonstrated by the variation of the $\bar{V}/V$ curves corresponding to zero radiation damping in *Figure 6*. Further, for small values of m, which implies either small bridge mass or relatively stiff soil conditions, a greater reduction in shear is obtained. Third, for slender piers on stiff soil conditions the effect of m on $\bar{V}/V$ is small and can be neglected. Finally, from the variation of $\bar{V}/V$ depicted in *Figures 6b* and *c*, it can be inferred that the reduction on base shear due to SSI is more significant in slender piers placed on stiff soil conditions.

Conclusions

This work studies the effects of soil-structure interaction on the longitudinal response of bridge piers to seismic excitations acting in the horizontal direction.

The study is based on a simple structure-soil idealization which, however, incorporates the most important features of soil-structure interaction. The model used for the bridge-soil system to a horizontal excitation has three degrees of freedom. An equivalent single-degree-of-freedom system is derived through dynamic equilibrium considerations that secure almost identical response amplitudes in both the multi-degree-of-freedom and the corresponding equivalent models. A thorough study of the equivalent single-degree-of-freedom system has led to: (1) an assessment of the effect of SSI on short span bridge-soil system; and (2) evaluation of the shear at the pier base including SSI with an easy-to-use procedure that can be implemented in preliminary bridge designs.

Cases in which SSI needs to be included in bridge design are identified and ways to take advantage of SSI in order to enhance safety and reduce design costs are recommended.

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Notation

A	cross-sectional area of pier
a	radius of circular foundation
C_h	horizontal viscous damping coefficient for radiation soil damping
C_r	rocking viscous damping coefficient for radiation soil damping
C_s	shear wave velocity for the soil
E	Young's modulus for the pier
G	soil shear modulus
h	height of the pier
$\bar{h}$	slenderness ratio
I	moment of inertia about the weak axis of the pier
$\bar{\zeta}$	damping ratio of equivalent system
ζ_h	damping ratio of viscous soil damping for lateral displacement
ζ	damping ratio of hysteretic soil damping
ζ_r	damping ratio of viscous soil damping for rocking motion
k	flexural stiffness of pier
k_h	horizontal stiffness of soil medium
k_r	rocking stiffness of soil medium
m	mass of bridge deck corresponding to one pier

M_r	moment at base of pier
P_h	horizontal force at the base of a pier
t	time variable
T	fundamental period of fixed base pier
$\bar{T}$	fundamental period of equivalent system
u	relative lateral displacement of bridge deck
u_g	lateral ground displacement
$\bar{u}_g$	lateral ground displacement of equivalent system
u_o	relative lateral displacement of pier base
u_t	total lateral displacement
V	base shear of fixed base pier
$\bar{V}$	base shear of equivalent system
γ	Poisson's ratio for the soil
θ	rotation angle
ρ	soil mass density
$\omega_s, \omega_h, \omega_r$	circular frequencies pertaining to a fixed base pier