Uplifting-sliding response of flexible structures to seismic loads

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This study presents the development of a BEM-FEM methodology to analyze flexible structures on an elastic halfspace allowed to simultaneously uplift and slide under seismic excitations. The methodology combines the Boundary Element Method (BEM) applied to model the semi-infinite soil medium and the Finite Element Method (FEM) to model the foundation and the super-structure. The two methods are combined with appropriate force equilibrium and displacement compatibility requirements through the utilization of FEM interface elements at the foundation-soil interface. All four modes of interface deformations, i.e., stick, debonding, rebonding and sliding, are accounted for to accurately simulate uplift and sliding of the foundation-structure system from the underlying soil medium. The sliding mode of deformation is associated with Coulomb friction at the soil-structure interface. The methodology is employed to investigate the response of a nuclear containment structure subjected to the El Centra earthquake of 1940. The results revealed that the base shear reduces significantly if the structure is allowed to slide. Further, parametric studies for various values of the friction coefficient are conducted in order to investigate the structure response under varying friction conditions.

Key Words: boundary elements, Coulomb friction, finite elements, flexible structures, half-plane, interface elements, seismic loads, sliding, uplifting.

INTRODUCTION

Several researchers have investigated the effects of uplift on the response of buildings, since Housner[1] pointed out the beneficial effects of uplift on structures subjected to earthquake loadings. The works reported in the literature can be divided according to their methodologies in the following three categories: **Category 1** – Employment of discrete systems to model the structure placed on a rigid soil. Such an approximation of the soil behaviour provides a convenient simplification when emphasis is placed on the investigation of the super-structure's dynamic response. However, it rules out the consideration of soil structure interaction in the system response[2–5]. **Category 2** – Use of springs and dashpots with frequency independent stiffness and damping properties to simulate the soil behaviour[6–10]. The most popular models are the 'Winkler' and its improved version, the 'Winkler-Voigt' foundation which places viscous dampers in parallel with the elastic springs to provide the subgrade reaction[11]. The primary justifications for the popularity of these models are their simplicity and accumulated data from practice. **Category 3** – Employment of the Finite Difference Method (FDM) or the Finite Element Method to model both the structure and the soil[12–15]. Both methods can effectively treat nonlinearities or sharply varying properties associated with the soil. It should be noted that the former method can not effectively model general geometric configurations. Both the FDM and the FEM introduce 'box effects'; that is, reflection of waves from artificial soil boundaries which in turn lead to spurious results. Box effects can be mitigated if 'non-reflecting' or consistent boundaries are used to alleviate the errors from the wave reflections.[16]

Even though numerous studies have addressed soil-structure interaction (SSI) under bilateral contact conditions, a very limited number of studies have considered the more complicated nonlinear SSI for unilateral conditions[17–24]. Using a linear finite element formulation and quadratic optimization algorithms, Talaslidis and Panagiotopoulos[17] studied a class of dynamic unilateral problems. Wolf and Obernhuber[18] developed a weighted residual technique utilizing Green's functions to study the partial uplift of the basement of a structure. Their procedure combined a flexibility formulation for the contribution of the soil using dynamic-flexibility coefficients in the time domain with a direct stiffness method for the structure. Antes and Steinfeld[16] determined the response of a dam allowed to uplift from the underlying soil medium subjected to externally applied impulsive vertical and horizontal loads. In their analysis, both the dam and the soil medium were modeled with a time domain Boundary Element Method (BEM). Employing a direct time domain BEM-FEM approach, Karabalis and Gaitanos[20] obtained the response of massless rigid or flexible surface foundations allowed to uplift. In their study, use was made of interface elements to evaluate the response of a square foundation subjected to vertical forces and moments. Kobori *et al.*[21], developed an analytical iterative procedure to determine the response of a shear building on foundations allowed to uplift. Their method involves the evaluation of nonlinear correction forces due to uplift in the time domain and a pseudo-linear response calculation in the frequency domain, considering nonlinear forces as equivalent accelerations applied on the foundation. Hillmer and Schmid[22] obtained the response of a building allowed to uplift utilizing a BEM-FEM formulation in the Laplace domain. Mendelsohn and Doong[23] used a time domain BEM to study the dynamic frictional contact of two elastic bodies under SH wave motion. The approach considered in this work extends the hybrid time domain BEM-FEM approach of Patel and Spyrakos[24] delaing only with basemat uplift to the more general case that includes basemat uplift and sliding with friction. The basic hybrid time domain BEM-FEM methodology was first developed by Spryakos and Beskos[25] and Karabalis and Beskos[26], who studied the dynamic response of two- and three-dimensional foundations in complete bond with the soil. In their works the BEM-FEM formulation employed the BEM to model semi-infinite soil media and the Finite Element Method (FEM) to model the finite domain of the structure and the foundation. The BEM is particularly well suited to model soil domains because of its ability to automatically account for radiation conditions at infinity, and reduces the spatial dimensions of the problem by one[27]. Further, the work presented herein incorporates FEM interface elements to model the foundation-soil contact area for the investigation of the nonlinearities arising from the soil-foundation separation and sliding. The BEM-FEM model is employed to investigate the response of a representative problem of a nuclear containment building subjected to the EL Centro earthquake of 1940. The response of the structure under conditions of uplift alone and that of uplift and sliding occurring simultaneously are obtained. Several values of frictional coefficient commonly used to represent the sliding of structure on soil are considered.

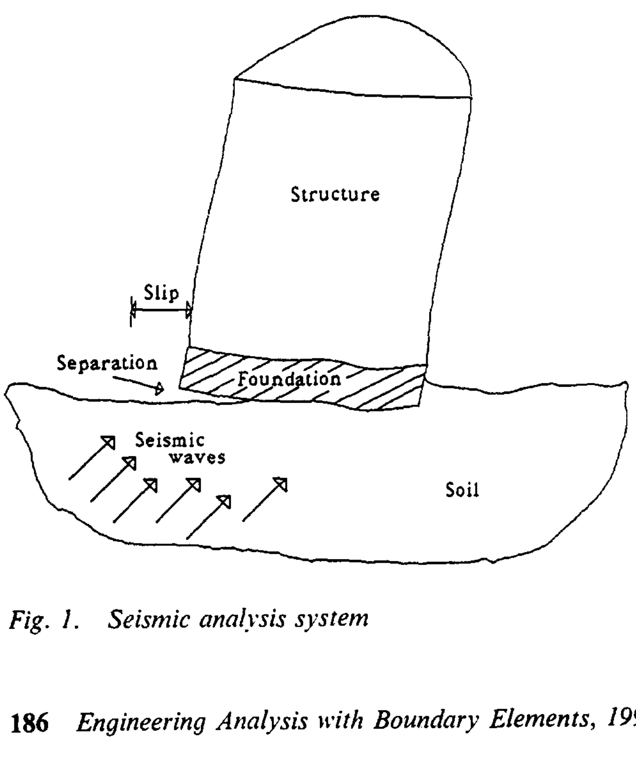


Fig. 1. Seismic analysis system

FORMULATION

The sequence of presenting the hybrid time domain BEM-FEM formulation, as can be appied to elastodynamic problems, consists of three steps. Namely, the development of the integral equations corresponding to Navier-Cauchy equations and boundary conditions of an infinite linear elastic medium; the development of the differential equations and boundary-initial conditions associated with the finite domain; and the numerical treatment of the governing integral and differential equations with the aid of the BEM and FEM, respectively. Herein the infinite medium is the soil, while the system of finite dimensions is the overlying foundation-structure system as shown in Fig. 1. Also dipicted in the same figure are the uplift and sliding deformations studied in this work. Shown in Fig. 2 are the FEM and BEM discretizations of the finite structure and the semi-infinite soil medium, respectively, along with FEM interface elements utilized to simulate the uplift and sliding of the foundation-structure system supported by the soil medium.

Assuming linear elastic behaviour, the response of homogeneous soil is governed by the Navier-Cauchy equations

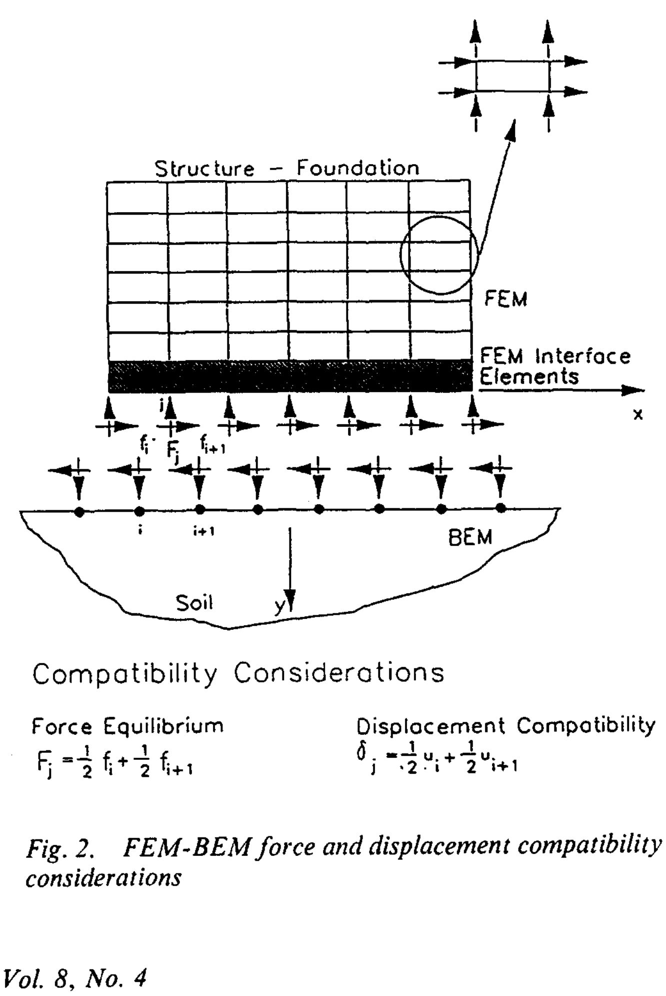


Fig. 2. FEM-BEM force and displacement compatibility considerations

$$(c_1^2 - c_2^2)u_{j,ji} + c_2^2 u_{i,jj} - \ddot{u}_i = -\frac{1}{\rho} b_i$$
$$(i, j = 1, 2) \tag{1}$$

where dots and commas indicate time and space differentiation, respectively, b_i are the body forces, ρ denotes the mass density of the soil medium, and c_1 and c_2 are the dilational and shear wave velocities, respectively.

The fundamental solution $U_i^{t}(x, \xi, t')$ of the Navier-Cauchy equations expressing the response of an infinite medium at time t' to a unit impulse force under conditions of plane strain, as given by Eringen and Suhubi[28] as

$$U_i^{(i)}(x, \xi, t') = \frac{1}{2\pi\rho} \left\{ \frac{1}{c_1} \frac{H(c_1 t' - r)}{r^2} \left[\frac{2c_1^2 t'^2 - r^2}{R_1} r_{,i} r_{,j} - R_1 \delta_{ij} \right] - \frac{1}{c_2} \frac{H(c_2 t' - r)}{r^2} \left[\frac{2c_2^2 t'^2 - r^2}{R_2} r_{,i} r_{,j} - \left(R_2 + \frac{r^2}{R_2} \right) \delta_{ij} \right] \right\} \quad (i, j = 1, 2) \tag{2}$$

$$R_i = \sqrt{c_i^2 t'^2 - r^2}$$
$$t' = t - \tau$$

where H is the Heaviside function and $r = |x - \xi|$ denotes the distance between a 'source' point, ξ, and a 'field' point x.

Assuming zero initial conditions and zero body forces, the integral identity corresponding to the Navier-Cauchy equations (1) and prescribed boundary conditions at the soil surface can be given by the equations[28]:

$$\frac{1}{2} u_j(\xi, t) = \oint_T (U_j^{(i)} * t_{(\eta)j} - T_j^{(i)} * u_j) dT \tag{3}$$

where $t_{(\eta)j}$ are the surface tractions, the operation * denotes time convolution and the expressions for the fundamental tractions $T_j^{(i)}$ associated with the fundamental displacements $U_j^{(i)}$ are given in Ref. 16. It is not possible to obtain a closed form solution of equation (3) for general boundary conditions, thus, resort is made to a numerical treatment.

The numerical treatment of equation (3) is performed by discretizing the surface of the soil into a Q number of elements and employing a time stepping solution algorithm. The spatial variation of the displacements and tractions along each element are assumed to be constant over each time step representing time discretization. Incorporating the aforementioned approximations, equation (3) takes the discretized form:

$$c_{ij} u^{N_p} = \sum_{q=1}^{Q} \sum_{n=1}^{N} \left(\left[\int_{\Delta s} G_{ij}^{nq} ds \right] \{ t^{N-l+1} \} - \left[\int_{\Delta s} F_{ij}^{nq} ds \right] \{ u^{N-l+1} \} \right), \tag{4}$$

where u^{N_p} represents the displacement at the center of the element p at the time step N, $l = n - m + 1$, and G_{ij}^{nq} and F_{ij}^{nq} denote the discretized fundamental displacements and tractions, respectively. In evaluating the line integrals of the fundamental solution pair, singularities appear when p is equal to q in equation (4) for every time step N. A detailed evaluation of these singularities is presented in Ref. 16.

The other component of the soil-structure interaction system, the foundation-structure system, is modeled with constant strain rectangular finite elements as shown in Fig. 2. The system of simultaneous differential equations describing the response of the assemblage of the discretized foundation-soil is given by

$$[M]\{\ddot{\delta}\} + [C]\{\dot{\delta}\} + [K]\{\delta\} = \{F(t)\} \tag{5}$$

where $[M]$, $[C]$ and $[K]$ are the consistent mass, damping and stiffness matrices of the foundation-structure system, respectively, the vectors $\{\ddot{\delta}\}$, $\{\dot{\delta}\}$ and $\{\delta\}$ denote nodal accelerations, velocities and displacements, respectively, and $\{F(t)\}$ is the vector of the time dependent nodal forces pertaining to the ground excitations.

The geometric nonlinearities considered herein result from the time dependent variation of the contact area between the soil and the foundation when uplift and sliding occurs. To effectively treat these nonlinearities, thin layer four node bilinear rectangular interface elements are used to discretize the soil-foundation interface. The four modes of deformations that accurately simulate the uplift and sliding of the foundation basemat are the stick, debonding, rebonding and slip modes. An interface element is in the stick mode when there is no relative motion between the adjoining bodies and no tensile stresses are developed from the external disturbances. The stick mode of interface deformation is characterized by the conditions:

$$\sigma_n^t > 0 \tag{6}$$

and

$$S_r^t > \tau_s^t \tag{7}$$

where σ_n^t and τ_s^t are the normal and shear stresses at time t, and S_r^t is the allowable shear stress defined by the Coulomb dry-friction criterion as,

$$S_r^t = \mu \sigma_n^t \tag{8}$$

where μ denotes the coefficient of friction of the interface element. For the stick mode of deformation the interface elements are treated like any other regular FEM elements with material properties which are identical to those of the underlying soil media. The additional stiffness and inertia introduced in the system are negligible, considering the semi-infinite soil media laying underneath the interface elements. Separation or debonding takes place when the bodies open up due to constraints of unilateral conditions, prohibiting the development of tensile stresses as they are incompatible with the constitutive properties of the geologic materials. The Young's Modulus of elasticity is reduced for the interface element in the debonding mode according to

$$[D]_{\text{debonding}} = 0.001 \times [D]_{\text{stick}}, \tag{9}$$

where $[D]_{\text{stick}}$ is the material property matrix in the stick mode. This, in essence, creates a void element with very little stiffness. If the interface element in the debonding mode returns to the stick mode during subsequent loading, rebonding takes place. The phenomenon of sliding is modeled through the slip mode of interface deformation. Herein, the failure criterion considered for slip mode is that of Coulomb dry-friction; accordingly, the slip mode of interface deformation can be detected through the following conditions:

$$\sigma_n^t > 0 \tag{10}$$

and

$$S_r^t \leqslant \tau_s^t \tag{11}$$

The tractions due to friction, opposing the relative motion of the foundation are given by

$$F_f = \mu\sigma_n^t \operatorname{sgn}(\dot{u}_r). \tag{12}$$

where the sgn($\dot{u}$) function denotes that the friction force F_f always opposes the motion, that is, its direction is opposite to that of the relative velocity $\dot{u}$ interface node. Interface elements similar to the one described above have been successfully used to solve static as well as dynamic two-dimensional problems, where all domains have been discretized with the aid of FEM[29]. In this study, the formulation pertaining to the interface elements is derived separately and then combined with that of the foundation-structure system prior to enforcing the compatibility and equilibrium with the BEM modeling of the soil[16,19,24].

The aspect in which the interface elements differ from regular elements, as the ones used to model the super-structure, is their thickness ratio,

$$t_r = \frac{t_{\text{int}}}{t_{\text{neigh}}}, \tag{13}$$

where the subscripts int and neigh pertains to the interface and the neighbouring regular FEM element, respectively. The small value of thickness of the interface element is liable to cause numerical problems[30]; however, Pande and Sharma[31] suggest guidelines that can be followed to circumvent the numerical problems. In this study, a thickness ratio of 0.1 has led to convergent results. More information on the complete implementation of the interface elements in the BEM-FEM methodology can be found in Ref. 16.

The two approaches, BEM and FEM, are coupled appropriately by displacement compatibility and force equilibrium considerations at the soil-foundation interface. More specifically, the displacements of the BEM soil elements are evaluated at the midpoint, while the FEM displacements correspond to the FEM nodes as shown in Fig. 2. In order to introduce compatibility between the deformations of the interface element nodes and the deformations at the soil surface, the average displacement of an interface element node q is approximated by the mean value of the nodal displacements at the ends of the BEM element p which is in contact with the element q. Similarly, compatibility of forces can be established if each contact force applied at node q is approximated by the mean value of the two resultant forces associated with the contact stresses, that develop over two successive elements joined at their common node. Thus, for the whole interface region, the compatibility relationships can be expressed as

$$\{\delta\} = [T]\{u\} \tag{14}$$

$$\{F\} = [T]\{f\} \tag{15}$$

where $[T]$ is the transformation matrix composed of zero and 1/2 entries[25]. In view of equations (14) and (15), equation (4) of the BEM formulation can be expressed as:

$$[T]^T[\bar{B}] = [T]^T[B][T]\{\delta\}, \tag{16}$$

where

$$[B] = \frac{l}{2}[G^1]^{-1} \tag{17}$$

and

$$\begin{aligned}[\bar{B}] = l[G^1]^{-1}([G^2]\{t^{N-1}\} + [G^3]\{t^{N-2}\} + \dots \\ + [G^N]\{t^1\}) - ([H^2]\{u^{N-1}\} + [H^3]\{u^{N-2}\} + \dots \\ + [H^N]\{u^1\}),\end{aligned} \tag{18}$$

in which the superscripts denote the time steps at which the quantities are evaluated and 1 represents the length of the element. It should be noted that equation (18) represents the time convolution process stipulated in equation (4), and indicates that the matrix $[\bar{B}]$ depends on the complete time history prior to the time step at which it is evaluated. Combining the BEM and FEM formulations results in the set of non-linear equations governing the response of the soil-structure system and reading

$$[M_t]\{\ddot{\delta}\} + [C_t]\{\dot{\delta}\} + [\bar{K}_t]\{\delta\} = \{F(t)\} \tag{19}$$

where the subscript t denotes the time dependent nature of the matrices arising from the changing contact area at the soil-foundation interface; $[M_t]$ and $[C_t]$ are the mass and damping matrices, respectively, $[\bar{K}_t]$ is the equivalent stiffness matrix given by

$$[\bar{K}_t] = \begin{bmatrix} [K] + [T]^T[B_{cc}][T] & [T]^T[B_{ce}][T] \\ [T]^T[B_{ec}][T] & [T]^Y[B_{ee}][T] \end{bmatrix} \tag{20}$$

and the time dependent forcing function $F(t)$ can be evaluated from

$$F(t) = \begin{bmatrix} [T]^T[\bar{B}_{cc}][T] + [K] & [T]^T[\bar{B}_{ce}] \\ [T]^T[\bar{B}_{ec}][T] & [\bar{B}_{ee}] \end{bmatrix} \begin{Bmatrix} \{\delta_c^f\} \\ \{u_e^f\} \end{Bmatrix} \tag{21}$$

where the superscript f denotes the free-field, and subscripts c and e represent the degrees of freedom corresponding to the foundation-soil contact area and the external discretization on either side of the contact area, respectively. The nonlinear system of equations (19) is solved with the aid of the direct integration scheme. A flowchart describing the procedure followed for evaluation of the contact area and the structure response is shown in the Appendix. Initially the foundation and the soil are assumed to be in complete bond. For each time step an iterative procedure is employed to determine the size of the correct contact area. More specifically, if at the end of a given iteration the contact area is not the same as the one assumed at the beginning of the iteration, a new contact area is computed based on the displacements found in the current iteration. The procedure is repeated until convergence that renders the contact area is obtained.

NUMERICAL EXAMPLE

The previously described time domain BEM-FEM methodology is employed to obtain the response of a nuclear containment structure subjected to the horizontal component of the El Centro earthquake of 1940. This containment structure has been studied by Weidlinger Associates[15], who conducted an experimental and a FEM

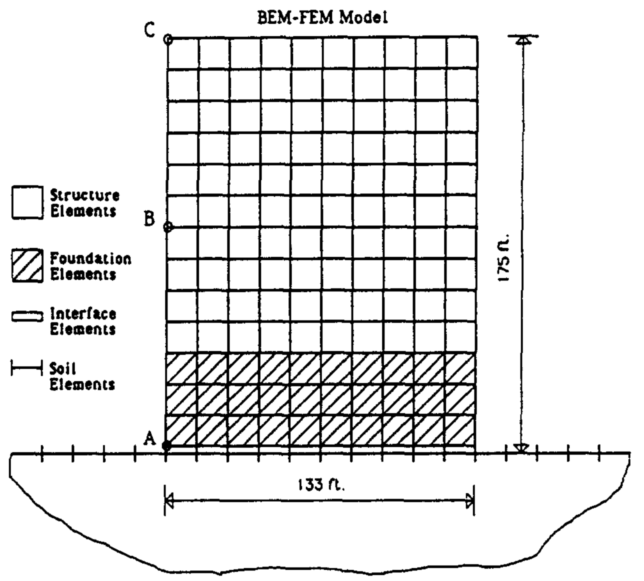


Fig. 3. FEM-BEM model of nuclear containment structure

soil-island analysis for blast loading arising from buried explosives. The soil is characterized by a shear wave velocity of 650 ft/sec and a pressure wave velocity of 1300 ft/sec, while a modulus of elasticity $E_f = 106560369$ lb/ft^2, a Poisson's ratio $\nu_f = 1/3$ and a mass density $\rho_f = 3.04348$ lb-sec^2/ft^2 are taken to be the material properties of the foundation elements. The super-structure is characterized by a modulus of elasticity $E = 106577648$ lb/ft^2, a Poisson's ratio $\nu = 1/3$ and a mass density $\rho = 0.59006$ lb-sec^2/ft^2. The equivalent two-dimensional plane strain model and its BEM-FEM discretization are shown in Fig. 3. The soil surface is discretized into 10 BEM elements at the soil-foundation contact area and into 5 BEM elements on either side of the foundation. This discretization has been selected so that the faster c_1 wave travels half the length of the BEM element during a time step, a requirement necessary to secure acceptable solution accuracy[32]. In order to further enhance the solution accuracy, the kernel matrices $[G]$ and $[H]$ in equation (18) have been evaluated on the basis of a further discretization of the twenty elements into five subelements per element. The structure-foundation system is discretized into 13 layers of 10 constant strain rectangular elements across the length of the foundation; hence, the soil-foundation interface is also discretized with the aid of 10 interface elements as shown in Fig. 3. No significant differences in the response time history were observed between the above discretization and a discretization consisting of 15 interface elements with a compatible super-structure FEM mesh, establishing a high level of confidence in the ability of the adopted discretization to accurately predict the correct contact area at the soil-foundation interface.

The absolute horizontal displacement response at location A of the building for two contact conditions, unilateral and unilateral contact plus sliding are plotted in Fig. 4. The results correspond to a frictional coefficient value of $\mu = 0.4$. The sliding appears to cause significant increase in the displacement compared to the unilateral contact conditions that include only uplift. This can be expected since during the slip mode of interface

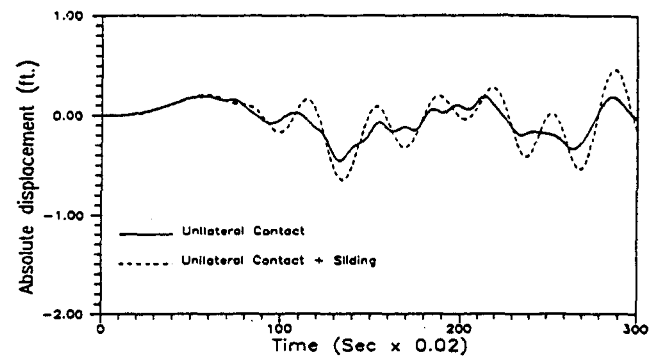


Fig. 4. Absolute horizontal displacement response at location A on the nuclear containment building

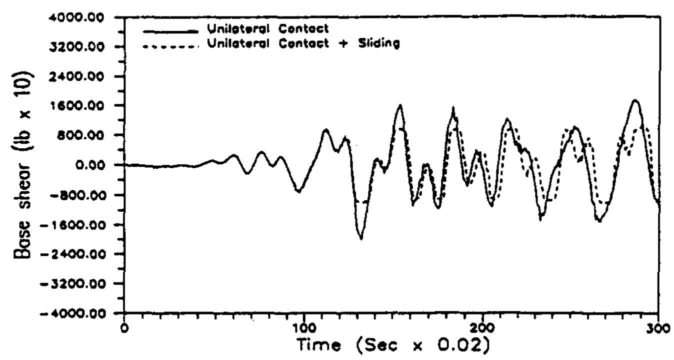


Fig. 5. Vertical displacement response at location A on the nuclear containment building

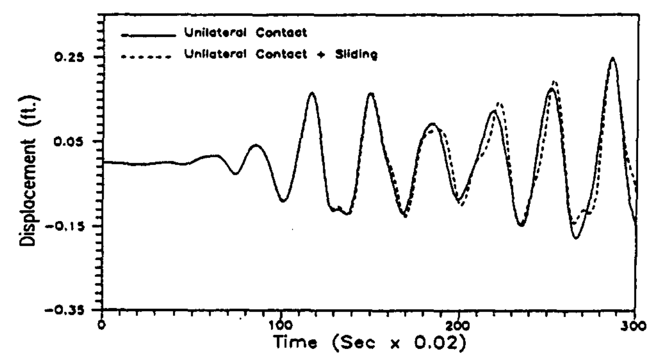


Fig. 6. Base shear response at the foundation-structure interface

deformation, the foundation-structure system will slide until the actions of the inertial forces in conjunction with the frictional forces reverses the slip motion. The vertical response history at A is shown in Fig. 5 for $\mu = 0.4$. The vertical displacement response is not affected significantly due to simultaneous unilateral contact and the sliding. However, the maximum displacement amplitudes are reduced slightly. Presented in Fig. 6, is the comparison of the base shear for unilateral contact including only uplift and unilateral contact with both uplift and sliding for $\mu = 0.4$. The results indicate a significant reduction of the maximum base shear in the response corresponding to the unilateral contact with uplift and sliding. Such results are expected because of the significant reductions in the relative displacements caused by sliding. The base shear values are listed for the various values of frictional

Table 1: Base shear for various values of frictional coefficient.			
Bilateral contact	Uplift only	Concurrent Uplift and Sliding	
Base Shear	Base Shear	Frictional coefficient	Base Shear
28970 lb	20130 lb	0.4	10233 lb
		0.3	6415 lb
		0.2	5958 lb

Table 1. Base shear for various values of frictional coefficient

coefficients in Table 1. A significant decrease in the maximum base shear is observed for $\mu = 0.3$ over $\mu = 0.4$. However, a little decrease is observed for $\mu = 0.2$, suggesting that no further gains in the reduction can be anticipated in lowering the value of μ. It also signifies that the base shear response amplitude obtained for the case of $\mu = 0.2$ is primarily due to the inertial forces in the superstructure and not due to the elastic deformations.

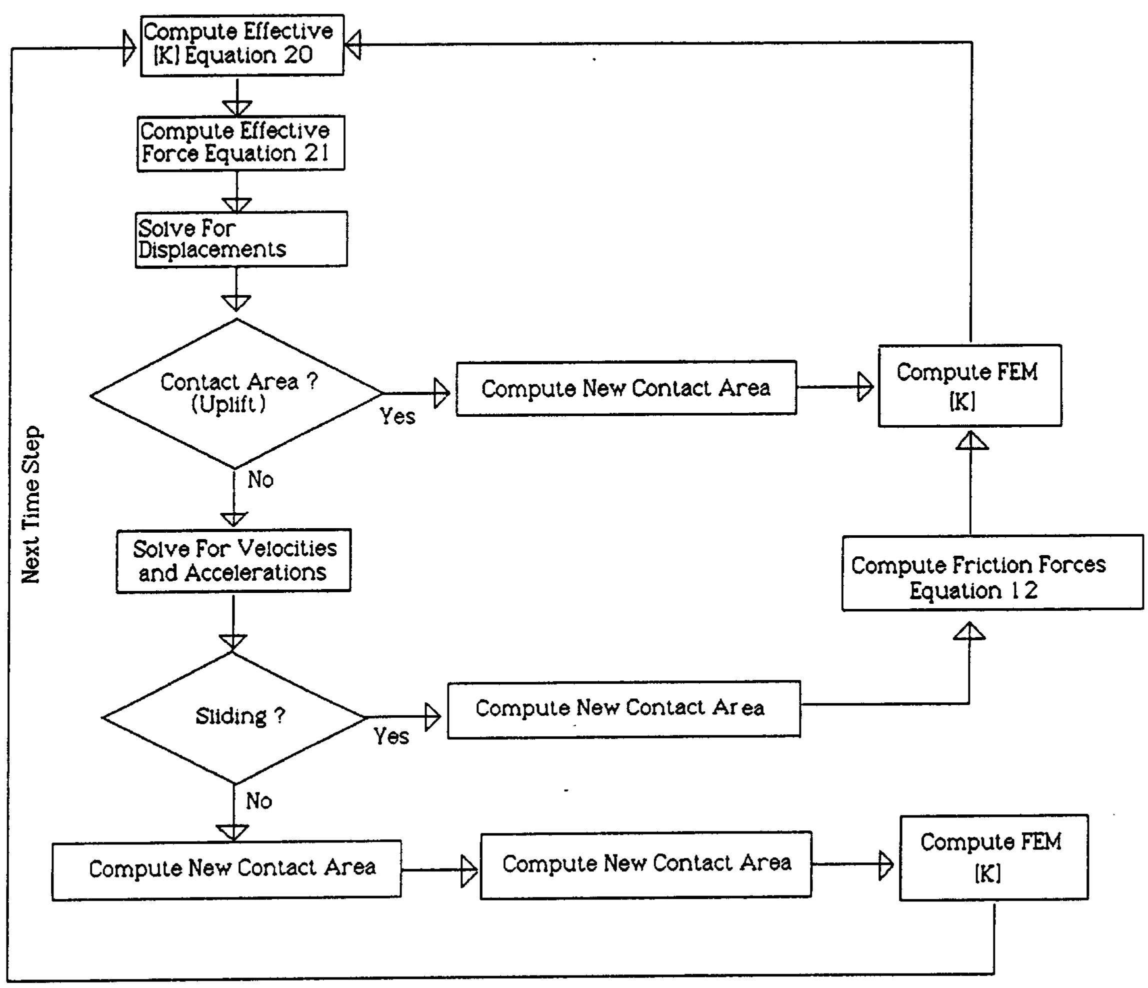
The results presented in Table 1 clearly demonstrate the significant reduction of base shear when sliding is allowed. Apparently, such a reduction combined with horizontal and uplift limitations imposed from service and architectural requirements, could lead to more cost effective design of structures in seismic zones.

CONCLUSIONS

A BEM-FEM methodology is developed for the analysis of SSI problems with consideration of the concurrent effects of basemat lift-off and sliding of the structure resting on soil. The boundary element method is applied to model the foundation-structure system. For seismic excitation, basemat lift-off is simulated by employing FEM interface elements, and sliding is incorporated in the model through Coulomb friction.

A numerical example of a nuclear containment structure subjected to the EL Centro earthquake of 1940 is presented. The solution is obtained for the case of uplift alone and uplift plus sliding. The results indicate a significant reduction in the base shear magnitude being accompanied by a substantial increase in the relative horizontal displacements at th foundation base, when uplift and sliding occur simultaneously.

The basemat lift-off and sliding of structures during severe earthquakes are likely to cause nonlinear soil behaviour in the vicinity of the soil-foundation contact area. The BEM-FEM methodology presented herein, could be extended to account for both geometric and material nonlinear behaviour in the vicinity of the soil-foundation contact area.



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