Available online at www.sciencedirect.com

SCIENCE DIRECT®

Computers and Structures 82 (2004) 2541–2550

Computers & Structures

www.elsevier.com/locate/compstruc

Dynamic analysis of flexible massive strip–foundations embedded in layered soils by hybrid BEM–FEM

C.C. Spyrakos [a,*], Chaojin Xu [b]

[a] Department of Civil Engineering, Laboratory of Earthquake Engineering, Polytechnic Campus, National Technical University of Athens, Zografos 15700 Athens, Greece

[b] Algor Inc., 150 Beta Dr., Pittsburgh PA 15238-2932, USA

Received 6 February 2003; accepted 6 May 2004
Available online 22 September 2004

Abstract

A study on the dynamic response of flexible massive strip–foundations embedded in layered soils is presented. The foundation is treated with a finite element formulation, while the difficulty in modeling the infinite extent of the soil is overcome by a boundary element formulation. The boundary element method is coupled with the finite element method by enforcing compatibility and equilibrium conditions at the soil–foundation interface. The accuracy of the proposed methodology is verified through comparison with results published for rigid foundations. Emphasis is also placed on parametric studies investigating the effects of salient factors such as foundation flexibility, mass and embedment.

Keywords: Soil–structure interaction; Strip–foundation; Finite element method; Boundary element method

1. Introduction

Studies involving dynamic soil–structure interaction (SSI) are rather complex because of the nonhomogeneity, nonlinearity and semi-infinite extent of the soil, as well as several difficulties in coupling the soil and the supported structure. The literature is rather extensive on the topic. A comprehensive review of the literature on soil–foundation interaction can be found in the papers by Gazetas [1], Antes and Spyrakos [2] and Spyrakos [3].

With limited analytical or empirical results to follow, design work in the past has been primarily based on rules-of-thumb methods. Modern widely accepted methods on dynamic analysis of foundations have been initiated by Hsieh [4] and Lysmer [5], and extended by Richart and Whitman [6] and Richart et al. [7]. In these methods, the vibrating massive foundation is represented by a set of "mass-spring-dashpots" oscillating with either frequency-dependent or frequency independent stiffness and damping coefficients, and the emphasis is placed on rigid foundations.

Simplified ground models have also been reported in the literature to obtain the response of either surface or embedded foundations. Representative are the works of Nogami and Chen [8], who developed closed form expressions to calculate the response of partially embedded rigid foundations, and Yongb et al. [9], who derived the impedance matrix for the relationship between displacements and external excitations of a rigid or flexible foundation embedded in a layered soil medium. Cone frustums have been used by Jaya and Prasad [10] to obtain the response of an embedded foundation in layered soil subjected to dynamic excitations. The significant differences between the response of flexible and rigid foundations are demonstrated through a 'rigid method' approach in a paper by Gucunski [11]. The

* Corresponding author. Tel.: +30-210-699-0041/42; fax: +30-210-699-0044.
E-mail address: spyrakos@hol.gr (C.C. Spyrakos).

0045-7949/$ - see front matter © 2004 Published by Elsevier Ltd.
doi:10.1016/j.compstruc.2004.05.002

vertical and rocking response of rigid foundations on a compressible nonhomogeneous half-space soil model has been studied by Vrettos [12] using a semi-analytical method.

Motivated by the need to examine the phenomena of soil–foundation interaction including foundation flexibility, nonhomogeneity and nonlinearity of soil, layered soil deposits, and possible partial separation between the foundation and the soil, more involved methods have been developed in the last three decades. The finite element method (FEM) has been one of the most widely used methods to solve soil–structure interaction problems. Its versatility as well as its shortcomings in analyzing media of infinite extent is well documented, e.g. Spyrakos [13]. Energy absorbing boundaries, such as transmitting boundaries, have also been used in FEM to overcome the problem of wave reflection and radiation on the boundaries of the soil domain (Murakami et al. [14], Kausel and Tassoulas [15], Day and Frazier [16], Basu and Chopra [17], Kim and Yun [18] and Zerfa and Loret [19]). As stated in the pioneering works of Kuhlemeyer [20], Kausel [21] and Lysmer et al. [22], the difficulty in applying FEM lies in selecting the proper transmitting boundaries. Use of half-space Green's functions for the soil medium coupled with the finite element method are one technique to avoid use of transmitting boundaries, e.g., the work of Bode at al. [23].

As an alternative method to FEM, the boundary element method (BEM), e.g., Wolf [24] and Ahmad and Rupani [25], as well as a combination of the FEM–BEM, the so-called hybrid FEM–BEM have also been used in SSI in the last two decades. In the FEM–BEM method, the FEM is utilized to model the foundation, and the BEM is employed to model the soil domain since it satisfies automatically the "far-field" boundary conditions associated with the semi-infinite soil domain; thus, eliminating the use of transmitting boundaries. Another advantage of BEM over the FEM is that it reduces the dimensions of the problem by one, and thus saves substantial modeling and processing time. The BEM has been applied to determine the response of both rigid and flexible foundations subjected to either static or dynamic loads. The response of embedded rigid and flexible foundations on an elastic half-space subjected to dynamic loads has been reported by Spyrakos and Beskos [26,27] and by Kokkinos and Spyrakos [28]. A comprehensive discussion on this topic is given by Spyrakos [13] for dynamic loads and seismic excitations. Analysis of rigid foundations on an elastic half-space allowed simultaneously to uplift and slide under seismic excitations has been performed by Patel and Spyrakos [29]. The behavior of rigid-massless foundations embedded in layered soils subjected to dynamic loads was studied by Ismail and Ahmad [30] and Ahmad and Bharadwaj [31]. The FEM–BEM has been employed by Yazdchi et al. [32] to study the response of dam–foundation interaction for seismic loads including the effects of pre-seismic loads such as water pressure and self-weight of the dam. A FEM–BEM formulation has also been used by Kim et al. [33,34] to study the response of a surface foundation and underground structures on multi-layered soil media subjected to incident wave excitations. It should be pointed out that the literature is rather extensive on the subject and only representative works are reported in this introduction.

In this work, the methodology of Kokkinos and Spyrakos [28] on massive surface flexible strip–foundations on an elastic half-space has been extended to study the response of massive flexible foundations embedded in layered soils subjected to externally applied loads. The effects of foundation flexibility and mass on the dynamic response are investigated in conjunction with the depth of embedment and soil layering.

2. Formulation and numerical treatment

The system consists of a flexible strip–foundation embedded in layered soil (see Fig. 1). The soil–structure interface is indicated as "c" and the top soil layer boundaries as "e", "c" and "1", respectively. The boundary "e" is the free surface and the boundary "1" is the contact interface with the soil layer [2]. The bottom layer has only the top boundary "$n-1$", since the bottom boundary "n" is extended to infinity. In general, a layer [k] has a top boundary "$k-1$" and a bottom boundary "k", where "$k-1$" is the interface between layers [$k-1$] and [k].

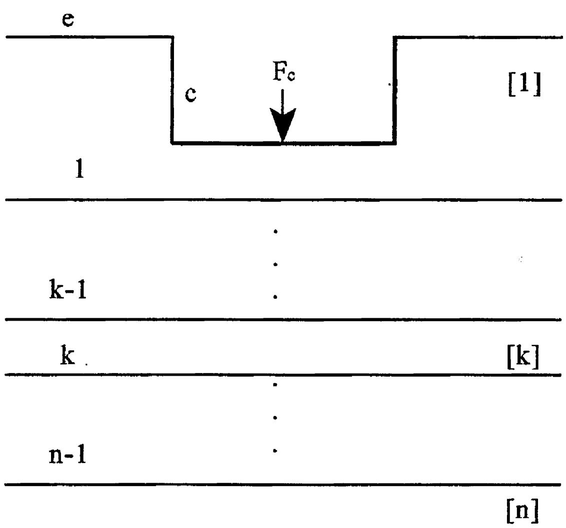

Fig. 1. Foundation embedded in layered soil.

3. BEM formulation for the soil

It is assumed that the soil layers are homogeneous, isotropic, and linear elastic and the displacements and strains are small. Under these assumptions, the governing equation for the soil is the well known Navier's equation. Expressed in terms of the transformed displacements in the frequency domain, Navier's equation is given by

$$(c_1^2 - c_2^2)U_{i,ij} + c_2^2 U_{j,ii} - k^2 U_j = 0 \tag{1}$$

where $k = i\omega$, ω is the circular frequency of the applied load, and c_1, c_2 are the P- and S-wave velocities, respectively. The c_1 and c_2 are given by

$$\begin{aligned} c_1 &= \sqrt{\frac{E_s(1-\nu_s)(1+2\xi i)}{\rho_s(1+\nu_s)(1-2\nu_s)}} \\ c_2 &= \sqrt{\frac{E_s(1+2\xi i)}{2\rho_s(1+\nu_s)}} \end{aligned} \tag{2}$$

in which E_s, ρ_s, ν_s are the modulus of elasticity, mass density, and Poisson's ratio, respectively, and ξ is introduced to include the effect of hysteretic damping [24]. In the frequency domain, the boundary conditions are expressed as

$$\begin{aligned} U_i(\vec{x};k) &= F_i(\vec{x};k) \quad \vec{x} \in \Gamma_1 \\ T_i(\vec{x};k) &= G_i(\vec{x};k) \quad \vec{x} \in \Gamma_2 \end{aligned} \tag{3}$$

where $F(\vec{x},k)$ and $G(\vec{x},k)$ are known displacements and tractions at the boundaries Γ_1 and Γ_2.

In this study, the BEM is used to numerically solve the boundary value problem defined by Eqs. (1)–(3). The starting point is the application of the reciprocal theorem, that is

$$\frac{1}{2}\delta_{ij}U_i(\vec{\xi},k) + \int_\Gamma T_{ij}^* U_j \mathrm{d}\Gamma = \int_\Gamma U_{ij}^* T_j \mathrm{d}\Gamma \tag{4}$$

where U_{ij}^*, T_{ij}^* are the fundamental solutions. Under plane strain, the expressions of U_{ij}^*, T_{ij}^* for a 2-D problem are given by Cruse and Rizzo [35], Spyrakos [13]

$$\begin{aligned} U_{ij}^*(\vec{\xi},\vec{x},k) &= \frac{1}{2\pi\rho c_2^2}(\psi\delta_{ij} - \chi r_{,i}r_{,j}) \\ T_{ij}^*(\vec{\xi},\vec{x},k) &= \frac{1}{2\pi}\left[\left(\frac{\mathrm{d}\psi}{\mathrm{d}r} - \frac{\chi}{r}\right)\left(\delta_{ij}\frac{\partial r}{\partial n} + r_{,j}n_i\right)\right. \\ &\quad - 2\frac{\chi}{r}\left(n_j r_{,i} - 2r_{,i}r_{,j}\frac{\partial r}{\partial n}\right) - 2\frac{\mathrm{d}\chi}{\mathrm{d}r}r_{,i}r_{,j}\frac{\partial r}{\partial n} \\ &\quad \left. + \left(\frac{c_1^2}{c_2^2} - 2\right)\left(\frac{\mathrm{d}\psi}{\mathrm{d}r} - \frac{\mathrm{d}\chi}{\mathrm{d}r} - \frac{\chi}{r}\right) r_{,i}n_{,j}\right] \end{aligned} \tag{5}$$

where

$$\begin{aligned} \psi &= K_0\left(\frac{kr}{c_2}\right) + \frac{c_2}{kr}\left[K_1\left(\frac{kr}{c_2}\right) - \frac{c_2}{c_1}K_1\left(\frac{kr}{c_1}\right)\right] \\ \chi &= K_2\left(\frac{kr}{c_2}\right) - \frac{c_2^2}{c_1^2}K_2\left(\frac{kr}{c_1}\right) \end{aligned} \tag{6}$$

in which K_0, K_1, and K_2 are modified Bessel functions of the second kind and order 0, 1, and 2, respectively, the vector x is the coordinate vector of any interested point in the domain and the vector ξ is the acting position of the Dirac's delta function, r is the distance between vectors x and ξ, and n is the outward normal direction [36].

The numerical solution of Eq. (4) requires discretization of the soil layers and soil–foundation interfaces. The boundary of a typical layer is divided into N boundary elements ($l = 1, 2, \ldots, N$). Using constant boundary elements and applying Dirac's delta function successively to the element nodes, one obtains a set of $2 \times N$ linear equations that relate the displacements to tractions, that is

$$[H]\{U^s\} = [G]\{T^s\} \tag{7}$$

where $[H]$ and $[G]$ are $2N \times 2N$ matrices that correspond to the fundamental solutions $\{U^*\}$ and $\{T^*\}$ of Eq. (5), $\{U^s\}$ and $\{T^s\}$ are the nodal displacement and traction vectors, respectively. A detailed description of the methodology can be found in Dominguez [36].

Replacing the traction vector $\{T^s\}$ in Eq. (7) by the nodal force vector, and after some matrix manipulations, the equilibrium equation in nodal quantities can be cast into the following form [37]:

$$[K^s]\{U^s\} = \{F^s\} \tag{8}$$

where

$$[K^s] = [L][G]^{-1}[H] \tag{9}$$

$$\{F^s\} = [L]\{T^s\} \tag{10}$$

in which

$$[L] = \mathrm{diag}\{l_1 l_1 \;\; l_2 l_2 \;\; \bullet\bullet\bullet \;\; l_N l_N\} \tag{11}$$

where l_i is the length of the i-th element.

4. FEM formulation for the strip–foundation

The thickness of the foundation is small compared to the other dimensions of the foundation and thus justifying the use of thin plate elements to model the foundation. Specifically, the Mindlin–Kirkorff plate theory is utilized in the FEM formulation for the strip–foundation [38,39]. This theory allows uncoupling of the governing equations for bending and axial deformations. For a plane strain problem, the equations in the frequency domain are reduced to the so-called Bernoulli–Euler beam problem. For bending deformation, the Bernoulli–Euler beam problem can be expressed as

$$\frac{d^4\widetilde{U}_y}{dy^4} - \lambda_2^4\widetilde{U}_y = 0 \tag{12}$$

Similarly, for axial deformation

$$\frac{d^2\widetilde{U}_x}{dx^2} + \lambda_1^2\widetilde{U}_x = 0 \tag{13}$$

where

$$\lambda_1^2 = \frac{\rho_f\omega^2}{E^*} \tag{14}$$

and:

$$\lambda_2^4 = \frac{\rho_f h\omega^2}{D_f} \tag{15}$$

in which D_f is the flexural rigidity of the plate defined by

$$D_f = \frac{E_f h^3}{12(1-v_f^2)} \tag{16}$$

where h is the thickness of the plate, and ρ_f, E_f, v_f are the modulus of elasticity, mass density, and Poisson's ratio, respectively. Notice that the bending stiffness EI and the unit mass per length m in the classic Bernoulli–Euler beam theory for bending are replaced by D_f and $\rho_f h$, respectively, and the modulus of elasticity E for axial deformation is replaced by $E^* = E_f/(1-v_f^2)$.

Following standard FEM procedures, the strip–foundation is divided into M elements and the equilibrium equation relating nodal forces to displacements in the frequency domain can be written in the form [40,41]

$$\begin{bmatrix} D_{uu} & D_{u\theta} \\ D_{\theta u} & D_{\theta\theta} \end{bmatrix} \begin{Bmatrix} U^f \\ \theta \end{Bmatrix} = \begin{Bmatrix} F^f \\ M \end{Bmatrix} \tag{17}$$

where the subscripts u and θ refer to nodal displacements and rotations, respectively. The expressions for the stiffness coefficients in Eq. (17) can be found in Ref. [42]. Through condensation of the rotational degrees-of-freedom, Eq. (17) can be rewritten in the following concise form:

$$\{F^f\} = [K^f]\{U^f\} \tag{18}$$

where

$$[K^f] = [D_{uu}] - [D_{u\theta}][D_{\theta\theta}]^{-1}[D_{\theta u}] \tag{19}$$

5. Coupling of BEM with FEM

The equilibrium equations for each sub-structure, i.e., the foundation and the soil layers, have been established. The system equation can now be obtained by satisfying compatibility and equilibrium conditions at the contact areas. At the interfaces, the sub-structures have equal displacements, and the forces from each sub-structure sum up to the total externally applied loads. Expressed in a vector form, they are

$$\begin{aligned} \{U_e^s\} &= \{U_e\} \\ \{U_c^s\} &= \{U_c^f\} = \{U_c\} \\ \{U_i^s\} &= \{U_{i-1}^s\} = \{U_i\} \end{aligned} \tag{20}$$

and

$$\begin{aligned} \{F_e^s\} &= \{0\} \\ \{F_c^s\} + \{F_c^f\} &= \{F_c\} \\ \{F_{i1}^s\} + \{F_i^s\} &= \{0\} \end{aligned} \tag{21}$$

where the displacement vectors without superscript indicate the common displacements at the interfaces, while "f" and "s" indicate structure and soil interfaces, respectively. The subscripts refer to the interfaces between the layers shown in Fig. 1.

Combining the equilibrium equations for each sub-structure, and employing the compatibility conditions at the interfaces, the equilibrium equation for the system is obtained [37]

$$[K]\{U\} = \{F\} \tag{22}$$

where

$$\{U\} = \{U_e \;\; U_c \;\; U_1 \;\; \bullet\bullet \;\; U_i \;\; \bullet\bullet \;\; U_{n-1}\}^T \tag{23}$$

$$\{F\} = \{0 \;\; F_c \;\; 0 \;\; \bullet\bullet \;\; 0 \;\; \bullet\bullet \;\; 0\}^T \tag{24}$$

and

$$[K] = \begin{vmatrix} K_{ee}^0 & K_{ec}^0 & K_{e1}^0 & & & & & & \\ K_{ce}^0 & K_{cc}^s + K_{cc}^0 & K_{c1}^0 & & & & & & \\ K_{1e}^0 & K_{1c}^0 & K_{11}^0 + K_{11}^1 & K_{12}^1 & & & & & \\ & & K_{21}^1 & \cdot & & & & & \\ & & & & \cdot & K_{i-1,i}^{i-1} & & & \\ & & & & K_{i,i-1}^{i-1} & K_{i,i}^{i-1} + K_{i,i}^i & K_{i,i+1}^i & & \\ & & & & & K_{i+1,i}^i & \cdot & & \\ & & & & & & & \cdot & K_{n-2,n-1}^{n-1} \\ & & & & & & & K_{n-1,n-2}^{n-1} & K_{n-1,n-1}^n \end{vmatrix} \tag{25}$$

6. Validation of the methodology

In order to eliminate the dependence of the results on the shear modulus of the soil and the amplitude of the load, the results are presented in a normalized from and are referred as normalized dynamic compliance. For example, the vertical normalized dynamic compliance of a foundation is expressed as:

$$\bar{f} = \frac{G_1 u}{P} = \mathrm{Re}[\bar{f}] + \mathrm{Im}[\bar{f}] \tag{26}$$

where G_1 is the shear modulus of the top soil layer, u is the displacement and P the amplitude of the external vertical load. The dynamic compliance is plotted against the dimensionless frequency $A_0 = \omega B/c_2$, where c_2 is the shear wave velocity of the top soil layer, ω is the frequency of the external load and B is half width of the foundation.

Since to the authors' knowledge, this is the first study of massive flexible strip–foundations on layered soils, comparisons were only possible for the limited case of rigid foundations that have been studied by other researchers, e.g., Gazetas [1] and Ahmad and Bharadwaj [31]. Two cases are investigated: In the first case, the results of a surface foundation on layered soil subjected to vertical loads are compared to those of Ismail and Ahmad [30]. The foundation is placed on a soil stratum laying over a half-space bedrock, see Fig. 2. The soil parameters are: Poisson's ratio $v_s = 0.33$, damping ratio $\xi_s = 5\%$, modulus of elasticity $E_s = 100$ MPa and mass density $\rho_s = 2000$ kg/m^3. The ratio of thickness of the soil stratum, H, to half width of the foundation, B, is $H/B = 2$. The material properties of the foundation are selected so that the foundation behaves almost like a rigid-massless foundation which is the case in the work of Ismail and Ahmad, that is, modulus of elasticity $E_f = 3 \times 10^7$ MPa, mass density $\rho_f = 0$, Poisson's ratio $v_f = 0.3$ and damping ratio $\xi_f = 5\%$. The vertical normalized real (Re) and imaginary (Im) parts of the dynamic compliance in Fig. 3 show an excellent agreement between the two works. In the second case, the absolute normalized dynamic compliance for a surface foundation subjected to a horizontal load is compared to the results given by Gazetas [1], see Fig. 4. The relatively small differences can be attributed to the fact that Gazetas' results have been reproduced from a graph provided in his paper.

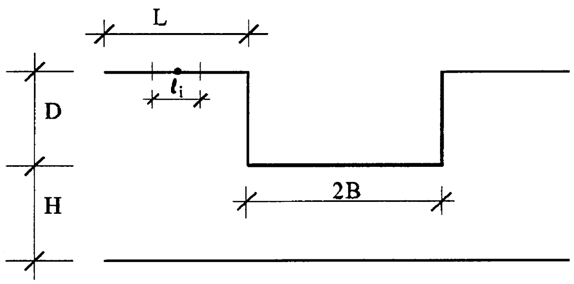

Fig. 2. Dimensions of the soil–foundation system.

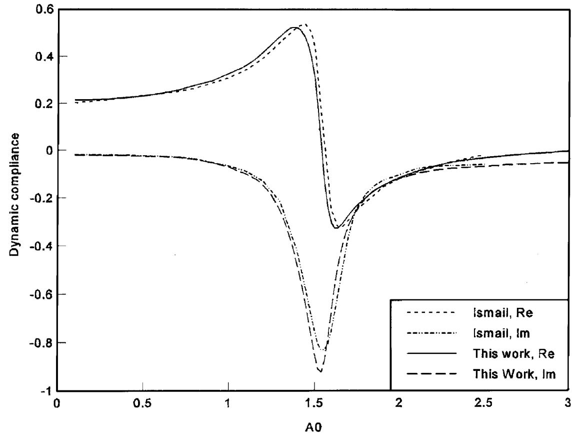

Fig. 3. Comparison for vertical response.

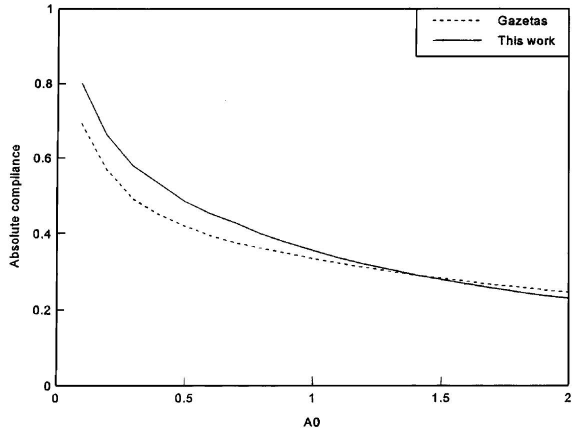

Fig. 4. Comparison for horizontal response.

In the BEM formulation using fundamental solutions for the infinite soil domain, two factors are of great importance in providing accurate results efficiently. The lengths of truncation distances along the boundaries L and the boundary element size l_i, see Fig. 2. The longer the truncation distances and the finer the discretization along the boundaries, the higher the solution accuracy, but the longer the computational time. Convergence studies for rigid foundations have been carried out by several researchers, e.g. Ahmad and Bharadwaj [31]. A transient fundamental solution that, combined with finite elements, solves transient half-space problems without discretization of the free surface has been developed by Guan et al. [43]. To select optimum truncation distances and discretization at the boundaries that maintain balance between accuracy and efficiency for flexible foundations, a series of convergence studies have been conducted by Xu [37], who has developed guidelines for computational efficiency and accuracy. Two significant recommendations extracted from Ref. [37] include: For vertical loads, a mesh with a truncation distance of twice the length of the surface standing wave from the edge of the foundation and a element length of one-tenth of the length of the surface standing wave provides good accuracy. Whereas for horizontal loads, the truncation distance must be doubled while keeping the same number of boundary elements in order to obtain accurate results efficiently.

7. Numerical results and discussions

The soil–structure system consists of a foundation with a width of $2B$ embedded in a soil layer with a depth H over a half-space bedrock, see Fig. 2. The depth of the embedment is D and the ratio of foundation width to thickness of the soil layer is $H/B = 2$. Through out this study, the material properties of the soil are: modulus of elasticity $E_s = 100$ MPa, mass density $\rho_s = 2000$ Kg/m^3, Poisson's ratio $v_s = 0.33$ and damping ratio $\xi_s = 0.05$. The Poisson's ratios $v_f = 0.3$ and damping ratio $\xi_f = 0.05$ are selected for the foundation. Two nondimensional parameters that characterize the soil–foundation system are introduced, they are: the relative stiffness K_r and relative mass density M_r, respectively, as defined in the paper by Kokkinos and Spyrakos [28]:

$$K_r = \frac{E_f h^3}{1 - v_f^2} \times \frac{1 + v_s}{E_s B^3}$$
$$M_r = \frac{\rho_f}{\rho_s} \qquad (27)$$

As elaborated by Kokinnos and Spyrakos [28], values of K_r 500, 5, 0.05 correspond to rigid, intermediate, and very soft foundations, respectively.

8. Effects of foundation flexibility

First, the effect of foundation flexibility on surface foundations is evaluated. Four relative stiffnesses, $K_r = 450$, 4.5, 0.45, 0.045 and a relative mass density $M_r = 1.25$ are considered for the vertical loads. For horizontal loads, the K_r values are: 45, 0.45, 0.045 and 0.0045 with $M_r = 0$. By selecting zero and nonzero values for M_r, the effect of foundation flexibility on both massless and massive foundations can be evaluated. The reason that smaller values of K_r are selected for horizontal loads is that the foundation behaves as rigid at a low relative stiffness as demonstrated in the results that follow. For both the vertical and the horizontal cases, the foundation with the largest and the smallest stiffness correspond to rigid and very soft foundations, respectively. Whereas the other two stiffness values correspond to flexible foundations.

The normalized dynamic compliances at the center of the foundation are plotted versus the nondimensional frequency A_0 in Figs. 5 and 6 for the vertical and horizontal loads applied at the center, respectively. These two figures clearly demonstrate that the effects of foundation flexibility are significant. For most of the frequency range studied, the general trend is that the smaller the relative stiffness, the larger the displacement. Also, the real part of the compliance is shifted uniformly upward, while the imaginary part of the compliance is shifted uniformly downward. This trend is similar for massless and massive foundations. Comparison between

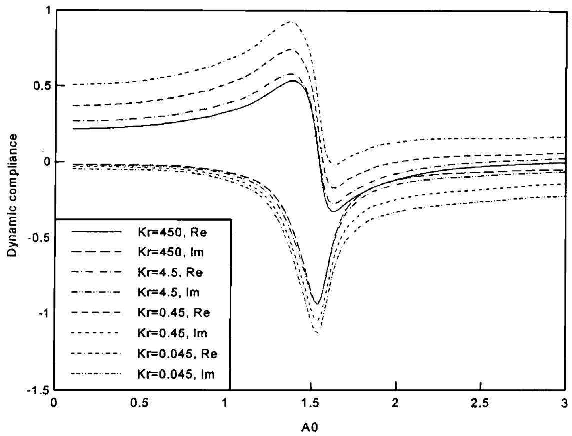

Fig. 5. Effect of foundation stiffness for surface foundation to vertical loads.

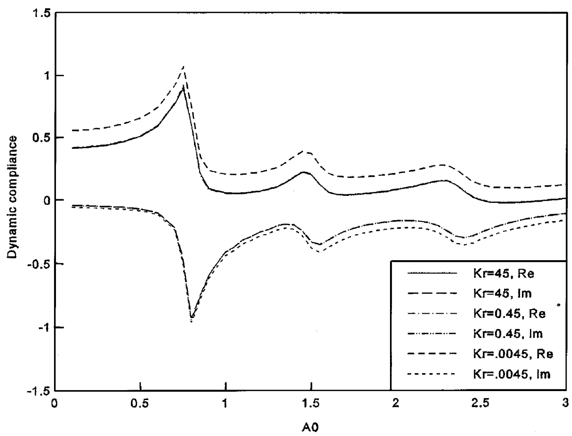

Fig. 6. Effect of foundation stiffness for surface foundation to horizontal loads.

Figs. 5 and 6 shows that the shift for vertical loads is larger than that for horizontal loads. Specifically, for a very soft foundation subjected to a vertical load, the compliance shows a shift of 150% from that of a rigid foundation at low frequency. However, for a very soft foundation under a horizontal load the shift is only 30% from the compliance of the rigid foundation at low frequency. Notice that the real part of the compliance is mostly affected, as it physically represents the stiffness of the soil–foundation system. The change of the imaginary part, expressing damping in the system, is less affected. Also notice that the natural frequencies of the system in horizontal motion are smaller than the ones in vertical motion. In fact, the fundamental frequency in Fig. 6 is about half the fundamental frequency in Fig. 5. It should be pointed out that in Fig. 6, the practical range of interest extends from 3×10^5 MPa ($K_r = 4.5$) to 3×10^4 MPa ($K_r = 0.45$) corresponding to steel and concrete foundations, respectively. For horizontal loads, the foundation stiffness depends mainly on the axial stiffness. As a result, the foundation becomes rigid at a relatively low value of the modulus of elasticity (see Fig. 6), i.e., 3×10^4 MPa, which is practically the modulus of elasticity of concrete. Whereas for vertical loads, the foundation still cannot be considered as rigid (see Fig. 6) for $E_f = 3 \times 10^5$ MPa, that corresponds to steel. These observations are of great importance in reassessing the validity of current practice, which as a norm ignores the effect of relative stiffness between the foundation and soil and considers the foundations either as rigid or very flexible.

The effect of foundation flexibility for embedded foundations is evaluated next. The ratio of embedment to half width of the foundation is $D/B = 1$. Three foundations with $M_r = 0$ and relative stiffness values, $K_r = 45$, 0.45, 0.0045, which could be defined as stiff, soft, and very soft are considered. The normalized dynamic compliances of the foundation subjected to vertical and horizontal loads are plotted in Figs. 7 and 8, respectively. Figs. 7 and 8 clearly demonstrate that the effects of foundation flexibility are significant for embedded foundations. Increase of foundation stiffness has a similar effect on the dynamic response of embedded foundations to that of surface foundations. As for surface foundations, the horizontal motion has a lower natural frequency than the vertical motion. However, the compliance is no longer uniformly shifted upward or downward as in the case of surface foundations. Comparisons between Figs. 5 and 7 as well as Figs. 6 and 8 show that the relative stiffness plays a greater role on modifying the response of embedded rather than surface foundations.

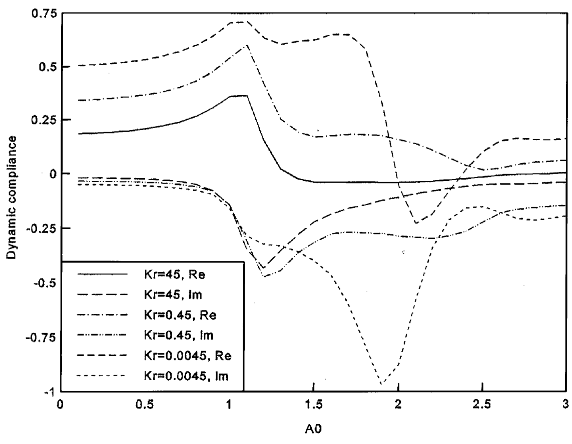

Fig. 7. Effect of foundation stiffness for embedded foundation to vertical loads.

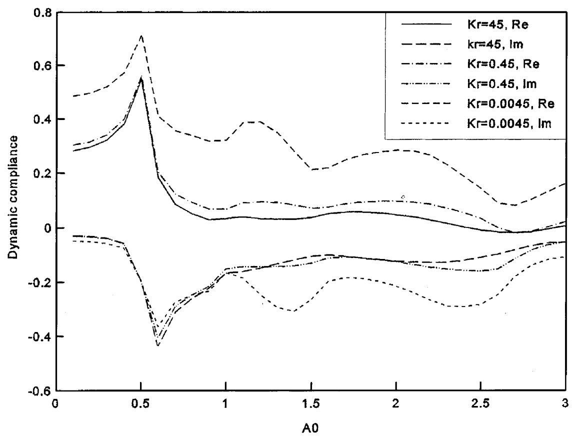

Fig. 8. Effect of foundation stiffness for embedded foundation to horizontal loads.

9. Effects of foundation mass

The effects of foundation mass on the response of surface foundations are investigated first. In evaluating the effects of foundation mass, the modulus of elasticity of the foundation is kept constant, while its mass density is varied. The relative foundation stiffness for the vertical load $K_r = 4.5$ corresponds to an intermediate foundation stiffness. For the horizontal load, the selected $K_r = 45$ represents a stiff foundation. Such selections of relative stiffness allow an assessment of the effects of foundation mass on both stiff and flexible foundation considered, i.e., $M_f = 0$, 1.25, 2.5, and 3.75.

Figs. 9 and 10 show the normalized dynamic compliances for the vertical and horizontal response at the center of the foundation subjected to vertical and horizontal loads, respectively. From Figs. 9 and 10 one may observe that by increasing the mass density of the foundation the natural frequency of the system

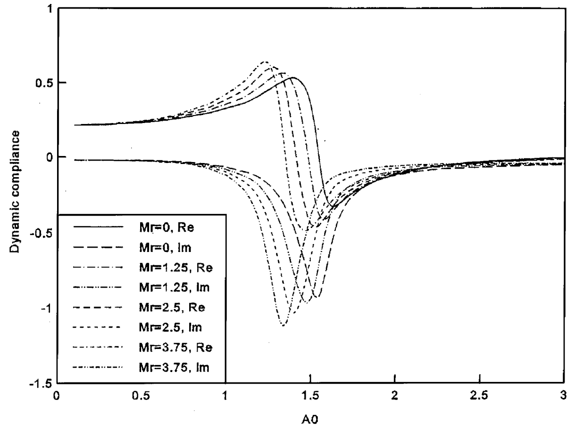

Fig. 9. Effect of foundation mass for surface foundation to vertical loads.

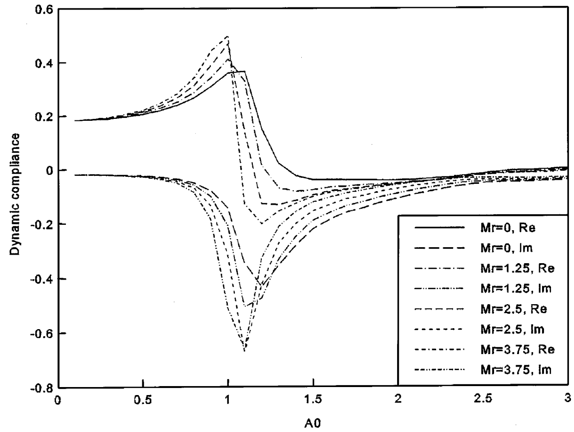

Fig. 11. Effect of foundation mass for embedded foundation to vertical loads.

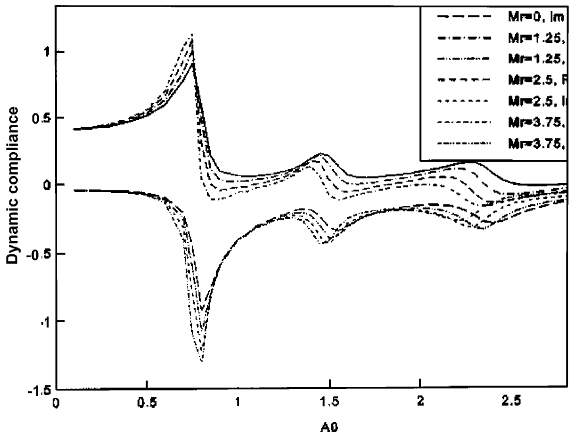

Fig. 10. Effect of foundation mass for surface foundation to horizontal loads.

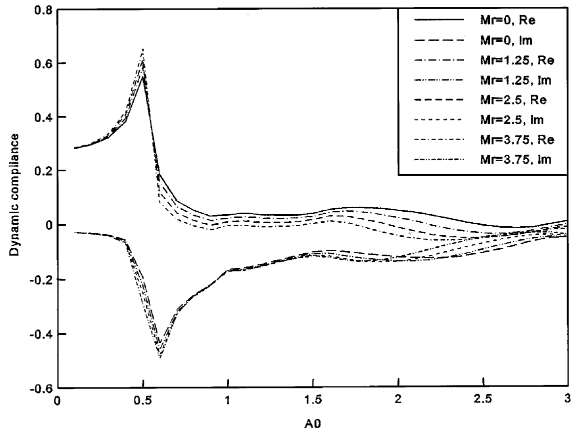

Fig. 12. Effect of foundation mass for embedded foundation to horizontal loads.

decreases. Also for frequencies close to the fundamental frequency of the soil–structure system, the greater the mass density, the larger is the corresponding displacement. However, foundation mass has a smaller effect on the response compared to that of foundation flexibility. Specifically, the difference in displacements between massive and massless foundation is within 20%. For frequencies away from the fundamental frequency, the influence of the relative mass density on the response is rather small. This trend is more distinct for vertical loads as depicted in Fig. 9.

For embedded foundations, the mass density has been varied for a relative stiffness of $K_r = 45$. The response of the system is obtained for four relative mass densities: $M_r = 0$, 1.25, 2.5, and 3.75. The normalized dynamic compliances for vertical and horizontal loads are drawn in Figs. 11 and 12, respectively. As can be observed in Figs. 11 and 12, no elaboration on the effects of embedment on the response is necessary, since they are identical to the ones made for surface foundations.

10. Effects of embedment

To investigate the effects of embedment, the foundation response for two different embedments, i.e., $D_1 = 2$ m and $D_2 = 4$ m is compared with the response for a surface foundation. In both cases, the foundations are massless with a relative stiffness of $K_r = 45$. The normalized dynamic compliances at the center of the foundation are shown in Figs. 13 and 14 for vertical and horizontal loads, respectively.

As can be observed in Figs. 13 and 14, the fundamental frequencies of the embedded foundation are de-

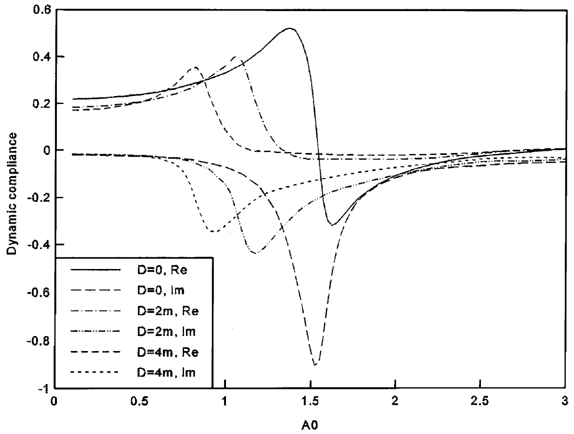

Fig. 13. Effect of foundation embedment for vertical loads.

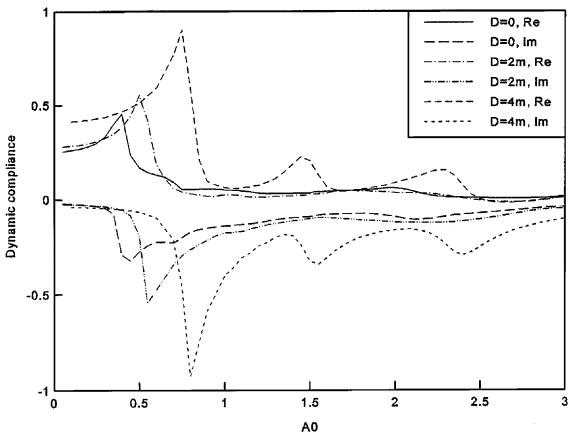

Fig. 14. Effect of foundation embedment for horizontal loads.

creased as compared to the surface foundation. This indicates that the effect of the additional inertia added from the embedment has more than counterbalanced the additional stiffness provided by the sidewalls of the foundation. Also, increasing the embedment depth of the foundation greatly reduces the displacement of the system. Notice that for a foundation with an embedment depth of 2 m, the displacement close to resonance is only 60% to that of a surface foundation. However, for an embedded foundation with an embedment depth of 4 m, the displacement close to resonance is only 40% to that of a surface foundation.

11. Conclusions

A numerical method has been presented for the dynamic analysis of massive flexible strip–foundations embedded in layered soils. The solution is based on a coupled BEM–FEM formulation where the FEM is employed to include the foundation flexibility and the BEM is used to overcome computational difficulties arising from the infinite extent of the soil. Recommendations for optimum truncation distances and element sizes are provided on the basis of convergence studies. The accuracy of the method is established through comparisons with published results on rigid foundations.

Parametric studies are conducted to investigate the effects of foundation-soil flexibility and mass as well as foundation embedment on the response. Foundation flexibility plays an important role on the dynamic response of foundations, especially for foundations subjected to vertical loads. For very soft foundations displacements can be tripled to that of rigid foundations. The displacements for moderately flexible foundations can be twice as large as those of rigid foundations at the low frequency range, i.e., frequencies of less than 10 cps. The studies show that because of the beneficial contribution of sidewalls, the displacement for embedded foundations is greatly reduced. The displacements are only 40% and 60% of that of the surface foundation for embedded foundations with an embedment of $D = 2$ m and $D = 4$ m, respectively.

References

[1] Gazetas G. Analysis of machine foundation vibrations: state of the art. Int J Soil Dyn Earthquake Eng 1983;2:2–42.

[2] Antes H, Spyrakos CC. Soil–structure interaction. In: Computer analysis and design of earthquake resistant structures: a handbook. Southampton: DE Beskos and SA Anagnostopoulos: Computational Mechanics Publications; 1997. p. 271–332 [Chapter 6].

[3] Spyrakos CC. Soil–structure interaction in practice. In: Oliveto G, Hall WS, editors. Chapter in boundary element methods for soil–structure interaction. London: Kluwer Academic Publishers; 2003. p. 235–75.

[4] Hsieh TK. Foundation vibrations. Proceedings of Institute of Civil Engineers 1962:22:211–26.

[5] Lysmer J. Vertical motion of rigid footings. Ph.D. thesis, University of Michigan, Ann Arbor, 1965.

[6] Richart FE, Whitman RV. Comparison of footing vibration tests with theory. J Soil Mech: Found Eng Div ASCE 1967;53:143–68.

[7] Richart FE, Wood RD, Hall JR. Vibrations of soils and foundations. New York: Prentice-Hall; 1970.

[8] Nogami T, Chen HS. Boundary differential equation method: simplified dynamic soil stiffnesses for embedded rigid foundations. Soil Dyn Earthquake Eng 2002;22(4): 323–34.

[9] Yongb Y, Zhanga R, Yub J. Motion of foundation on a layered soil medium—I. impedance characteristics. Soil Dyn Earthquake Eng 1997;16(5):295–306.

[10] Jaya KP, Prasad AM. Embedded foundation in layered soil under dynamic excitations. Soil Dyn Earthquake Eng 2002;22(6):485–98.
[11] Gucunski N. Rocking response of flexible circular foundations on layered media. Soil Dyn Earthquake Eng 1996;15(8):485–97.
[12] Vrettos C. Vertical and rocking impedances for rigid rectangular foundations on soils with bounded non-homogeneity. Earthquake Eng Struct Dyn 1999;28(12): 1525–40.
[13] Spyrakos CC. In: Manolis GD, Davis TG, editors. Boundary element techniques in geomechanics, Chapter 5. CMP and Elsevier; 1993.
[14] Murakami H, Shioya S, Yamada R, Luco JE. Transmitting boundary for time-harmonic elastodynamic of infinite domains. Int J Numer Meth Eng 1981;17:1697–716.
[15] Kausel E, Tassoulas JL. Transmitting boundaries: a close form comparison. Bull Seism Soc Am 1981;71:143–59.
[16] Day SM, Frazier GA. Seismic response of hemispherical foundation. J Eng Mech Div ASCE 1979;105:29–41.
[17] Basu U, Chopra AK. Numerical evaluation of the damping-solvent extraction method in the frequency domain. Earthquake Eng Struct Dyn 2002;31(6):1231–50.
[18] Kim DK, Yun C. Earthquake response analysis in the time domain for 2D soil–structure systems using analytical frequency-dependent infinite elements. Int J Numer Meth Eng 2003;58(12):1837–55.
[19] Zerfa Z, Loret B. A viscous boundary for transient analyses of saturated porous media. Earthquake Eng Struct Dyn 2003;33(1):89–110.
[20] Kuhlemeyer R. Vertical vibration of footings embedded in layered media. Ph.D. dissertation, University of California, Berkeley, 1969.
[21] Kausel E. Forced vibration of circular foundations on layered media. Research Report R74-11, MIT, 1974.
[22] Lysmer J, Udaka T, Seed HB, Hwang R. LUSH-A Computer program for complex response analysis of soil–structure systems. Report EERC 74-4, University of California, Berkeley, 1974.
[23] Bode C, Hirschauer R, Savidis SA. Soil–structure interaction in the time domain using halfspace Green's functions. Soil Dyn Earthquake Eng 2002;22(4):283–95.
[24] Wolf JP. Dynamic soil–structure interaction. Englewood Cliffs, NJ: Prentice-Hall, Inc.; 1984.
[25] Ahmad S, Rupani AK. Horizontal impedance of square foundation in layered soil. Soil Dyn Earthquake Eng 1999;18(1):59–69.
[26] Spyrakos CC, Beskos DE. Dynamic response of flexible strip–foundations by boundary element method. Int J Numer Meth Eng 1986;23:1547–65.
[27] Spyrakos CC, Beskos DE. Dynamic response of rigid-foundations by boundary and finite element method. Soil Dyn Earthquake Eng 1986;5:84–96.
[28] Kokkinos FT, Spyrakos CC. Dynamic analysis of flexible strip–foundations in the frequency domain. Comput Struct 1991;39(5):473–82.
[29] Patel PN, Spyrakos CC. Time domain BEM–FEM seismic analysis including basement lift-off. Eng Struct 1990;12(7):195–207.
[30] Ismail ASM, Ahmad S. Dynamic vertical compliance of strip foundations in layered soil. Earthquake Eng Struct Dyn 1989;18:933–50.
[31] Ahmad S, Bharadwaj A. Horizontal impedance of embedded strip foundations in layered soil. J Geotech Eng 1991;117:1021–41.
[32] Yazdchi M, Khalili N, Valliappan S. Dynamic soil–structure interaction analysis via coupled finite-element-boundary-element method. Soil Dyn Earthquake Eng 1999;18(7):499–517.
[33] Kim MK, Lim YM, Cho WY. Three dimensional dynamic response of surface foundation on layered half-space. Eng Struct 2001;23(11):1427–36.
[34] Kim MK, Lim YM, Rheel JW. Dynamic analysis of layered half planes by coupled finite and boundary elements. Eng Struct 2000;22(6):670–80.
[35] Cruse TV, Rizzo FJ. A direct formulation and numerical solution of the general transient elasto-dynamic problem. J Math Anal Appl 1968;22:244–59.
[36] Dominguez J. Boundary elements in dynamics. New York: CMP and Elsevier Applied Science; 1993.
[37] Xu CJ. Dynamic soil–structure–fluid analysis by hybrid FEM–BEM. Ph.D. dissertation, West Virginia University, Morgantown, WV, 1995.
[38] Timoshenko SP, Woinowsky-Krieger S. Theory of plates and shells. New York: McGraw-Hill; 1959.
[39] Ugural AC. Stresses in plates and shells. New York: McGraw-Hill; 1981.
[40] Paz M. Structural dynamics theory and computation. 3rd ed. New York: Van Nostrand Reinhold Company; 1991.
[41] Spyrakos CC, Raftoyiannis J. Linear and nonlinear finite element analysis in engineering practice. Pittsburgh, PA: Algor Publishing Division; 1997.
[42] Spyrakos CC, Beskos DE. Dynamic response of frameworks by Fourier transform. Comput Struct 1982;15:495–505.
[43] Guan F, Moore D, Spyrakos CC. Two dimensional transient fundamental solution due to suddenly applied load in a half-space. Soil Dyn Earthquake Eng 1998;17:269–77.