Overturning Stability Criteria for Flexible Structures to Earthquakes

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Abstract: Criteria for overturning stability of flexible structures such as chimneys and towers are developed. To the authors' knowledge all published studies arrive at overturning stability conclusions and criteria based on rigid body motion of systems. This is the first attempt to develop a simple design criterion that includes flexibility effects of slender structures to their overturning stability. The development is based on expressing the system deformation in terms of generalized coordinates. Examples and parametric studies demonstrate use of the criteria as well as the role that each one of the most significant geometric, inertial, and spectral parameters plays on the overturning stability of towers and chimneys. The procedure could also be useful to address the inverse problem that is to estimate the ground acceleration that caused the overturn of a slender structure.

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Introduction

Several forms of towers and chimneys have been built in earthquake areas, such as chimneys with supporting towers, guyed structures, and structurally combined multiple chimneys (Dowrick 1990). Towers and chimneys are among the geometrically simplest structures subjected to earthquake motions. However, they are more vulnerable to earthquake loads than the majority of the more redundant structural forms (Rumman 1967). Unlike building structures with high degree-of-redundancy, little reliance should be placed on the ductile behavior of chimneys and towers, since formation of one plastic hinge could lead to collapse, while inadequate design of their foundation could lead to local damage at the foundation level and even overturn.

Most design codes and research studies recommend use of dynamic analysis to estimate the distribution and magnitude of forces for the seismic design of both steel and reinforced concrete chimneys and towers (SEAOC 1967;Rinne 1970;ACI 1979;Berg 1989;Spyrakos 1995;CEN 1996). It is also recognized that, in many instances, seismic analysis of chimneys and towers should also include the effects of shear and bending deformations, soil–structure interactipn, P-delta as well as rotational inertia, since the contribution of all these effects could be significant (Watt et al. 1978; Luco 1986; Dowrick 1990; Spyrakos and Xu 1997). Furthermore, taking into account the supporting soil flexibility, foundation uplift should be also included in the analysis. A comprehensive review of studies on foundations in bilateral and unilateral contact before 1988 has been presented by Spyrakos (Spyrakos 1988). American Technology Council (ATC) document *ATC-40* (ATC 1996) also includes an extensive list of pertinent references in order to assist design engineers who would like to incorporate the effects of foundation uplift into their analysis. Finite element method–boundary element method formulations have also been developed to study elastodynamic problems involving partial loss of contact and sliding between elastic bodies (e.g., Patel and Spyrakos 1991;Wolf 1994). A representative experimental study of the seismic response of a reinforced concrete observation tower that has been carried out by Ganev et al. (1995) found that partial separation of the structure from the soil occurs under large dynamic loads, leading to changes in the predominant frequency of the system and its response.

A considerable number of studies on the overturn of tall slender structures as well as inverted pendulum type systems can be found in the literature (Housner 1963). Most studies have been spurred from either the surprisingly stable behavior of slender structures against overturning or the need to develop methods to prevent overturning and damage of furniture and equipment installed in buildings. Many researchers have also surveyed overturned bodies to estimate the seismic intensity of earthquakes. The use of overturned tombstones to deduce the seismic intensity has been a well-known practice. A thorough review of the up to 1980 literature on the overturning of bodies during earthquakes has been presented by Ishiyama (1980). To the authors' knowledge all published studies arrive at overturning stability conclusions and criteria based on rigid body motion of the analyzed systems. Little or no attention is placed on the flexibility effects of slender structures on their overturning stability. This issue, however, is of increasing importance because of the large number of slender structures built with greater frequency in areas of high seismicity.

This study is an attempt to address the aspect of overturning stability of towers, chimneys, and slender structures in general, as the loss of it could lead to failure of such structural systems

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subjected to strong ground motions. The work presents the development of criteria that can be used in conjunction with response spectra to assess the seismic vulnerability of slender structures to overturn. The criteria could also be used to estimate the seismic intensity by overturn of flexible structures.

Formulation of Basic Expressions

Sufficient results for the seismic analysis and design of slender structures, such as towers and chimneys, can be obtained with beam type models (Pinfold 1975;Berg 1989). For such a beam type structural system subjected to a seismic excitation, the equation of motion, neglecting shear deformations and rotatory inertia effects, can be expressed by

$$m\ddot{u} + c\dot{u} + EIu'''' = -m\ddot{u}_g \tag{1}$$

where m=mass per unit length; E=Young's modulus; I=second moment of area; c=equivalent viscous damping coefficient per unit length; and $\ddot{u}_g$=ground excitation. Furthermore, primes ($'$) indicate derivatives with respect to the beam axis y and overdots ($\cdot$) indicate derivatives with respect to time. For a single-degree-of-freedom approximation, the relative displacement u can be expressed in terms of space and time dependent functions as

$$u(y,t) = \phi(y)q(t) \tag{2}$$

where $\phi(y)$=normal mode shape and $q(t)$=generalized coordinate.

Although the higher modes may have a significant effect on the response of a tall tower structure, the study attempts to develop a simple design criterion for overturning stability of slender structures, based on a single mode domination of the dynamic characteristics of the structure. The normal shape $\phi(y)$ corresponding to the first mode can be evaluated by considering the boundary conditions at the ends of the beam. At the top, the tower is allowed to freely rotate and displace, while at the bottom the condition depends on the foundation type. Specifically, for a relatively small and flexible foundation the condition can be assumed to be free at the soil–foundation interface O_2O_1 (see Fig. 3), while for a rather large and stiff foundation supporting a flexible tower the condition can be estimated as fixed (see interface CD in Fig. 6). Note that both boundary conditions refer to the tower once it is uplifted. Assuming that one end of the beam is fixed while the other end is free to vibrate, the corresponding normal shape is (Craig 1981;Paz 1991)

$$\phi(y) = (\cosh 1.8751y/l - \cos 1.8751y/l) - \sigma(\sinh 1.8751y/l - \sin 1.8751y/l) \tag{3}$$

where

$$\sigma = \frac{\cos 1.8751 + \cosh 1.8751}{\sin 1.8751 + \sinh 1.8751} = 0.734$$

Similarly, the first mode shape for a beam with both ends free is given by (Pinfold 1975)

$$\phi(y) = (\cosh 4.73y/l + \cos 4.73y/l) - \sigma(\sinh 4.73y/l + \sin 4.73y/l) \tag{4}$$

where

$$\sigma = \frac{\cosh 4.73 - \cos 4.73}{\sinh 4.73 - \sin 4.73} = 0.9825$$

Substituting Eq. (2) into Eq. (1), multiplying the resulting equation by $\phi(y)$, and then integrating over the beam length l, results in

$$m^*\ddot{q}(t) + c^*\dot{q}(t) + k^*q(t) = -m\phi_I\ddot{u}_g(t) \tag{5}$$

where

$$m^* = \int_0^1 m\phi^2(y)dy$$

$$c^* = \int_0^1 c\phi^2(y)dy$$

$$k^* = \int_0^1 E\dot{I}\phi''(y)^2dy$$

$$\phi_I = \int_0^1 \phi(y)dy \tag{6}$$

Ignoring the sign of the effective earthquake force, i.e., the term on the right hand side of Eq. (5), Eq. (5) can be written as

$$\ddot{q}(t) + 2\xi\omega\dot{q}(t) + \omega^2q(t) = \frac{m\phi_I}{m^*}\ddot{u}_g(t) \tag{7}$$

where ω=fundamental natural frequency of the system and ξ=first modal equivalent damping ratio.

The solution of Eq. (7) can be expressed in terms of Duhamel's integral expression for low-damped systems, as follows (Clough and Penzien 1993):

$$q(t) = \frac{m\varphi_I}{m^*} \cdot \frac{1}{\omega} \cdot \Gamma(t) \tag{8}$$

where

$$\Gamma(t) = \int_0^t \ddot{u}_g(t)\exp[-\xi\omega(t-\tau)]\sin\omega(t-\tau)d\tau \tag{9}$$

The maximum absolute value over the entire earthquake history $|\Gamma(t)|_{\max}$ of the earthquake response integral $\Gamma(t)$ in Eq. (9) is the pseudovelocity spectral response S_v (Clough and Penzien 1993). Thus, the generalized coordinate q can be expressed as

$$q = \frac{m\phi_I}{m^*\omega}S_v \tag{10}$$

The resistant base shear force V_b and the overturning base moment M_b are given by (Clough and Penzien 1993; Chopra 1995)

$$V_b = \frac{(m\phi_I)^2}{m^*}\omega S_v$$

$$M_b = \frac{m\phi_I}{m^*}\omega S_v \int_0^1 my\phi(y)dy \tag{11}$$

It is noted that S_v depends on the ground motion history as well as the fundamental frequency of vibration and damping of the structure.

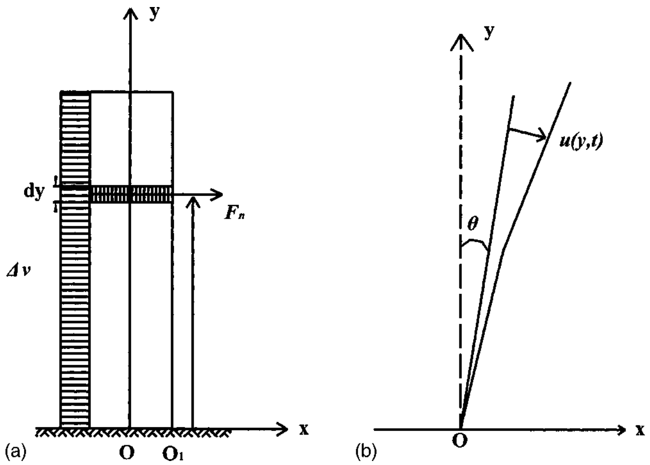

Fig. 1. Flexible beam configuration: (a) Tall slender tower; and (b) beam model with two degrees-of-freedom

Derivation of Overturning Criterion

The configuration of a flexible slender structure is shown in Fig. 1. The motion of the beam type structure is expressed in terms of two generalized coordinates: the rotational angle θ associated with the rigid body portion of the motion and the translational deformation $u(y,t)$ related to flexural deformed shape.

Consider a ground acceleration record composed of a sequence of N discrete step changes $\pm\Delta v$ in ground velocity that are randomly distributed and have equal probability of being positive or negative in sign. This represents an idealized earthquake ground motion with a constant undamped average velocity response spectrum (Housner 1963). The effect of such ground motion on structures is the same as if the ground were at rest and impulsive inertial forces F_n were acting through the center of mass (see Fig. 1) where

$$F_n \Delta t = \pm m \Delta v \tag{12}$$

The kinetic energy per unit length is given by

$$\overline{\mathrm{KE}} = \tfrac{1}{2} m y^2 \dot{\theta}^2 + \tfrac{1}{2} m \phi^2 \dot{q}^2 + m y \phi \dot{q} \dot{\theta} \tag{13}$$

The inertial forces per unit length with respect to θ and q are expressed by

$$\frac{\partial}{\partial t}\frac{\partial \overline{\mathrm{KE}}}{\partial \dot{\theta}} = m y^2 \ddot{\theta} + m y \phi \ddot{q}$$

$$\frac{\partial}{\partial t}\frac{\partial \overline{\mathrm{KE}}}{\partial \dot{q}} = m y \phi \ddot{\theta} + m \phi^2 \ddot{q} \tag{14}$$

Force equilibrium for a change of the external impulse velocity and inertial forces is given by

$$m y^2 \frac{\Delta \dot{\theta}}{\Delta t} + m y \phi \frac{\Delta \dot{q}}{\Delta t} = F_n y$$

$$m y \phi \frac{\Delta \dot{\theta}}{\Delta t} + m \phi^2 \frac{\Delta \dot{q}}{\Delta t} = F_n \tag{15}$$

By substituting Eq. (12) into Eqs. (15) and integrating along the overall length, one arrives at the equilibrium equations of the whole beam

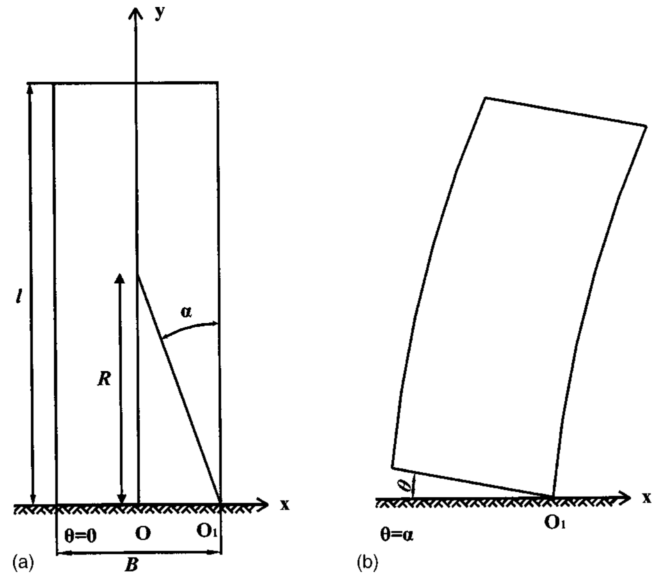

Fig. 2. Overturning about bottom corner (a) Initial position ($\theta=0$); and (b) final position ($\theta=\alpha$)

$$I_0^* \Delta \dot{\theta} + m \phi_{II} \Delta \dot{q} = \tfrac{1}{2} m l^2 \Delta v$$

$$m \phi_{II} \Delta \dot{\theta} + m^* \Delta \dot{q} = m l \Delta v \tag{16}$$

where

$$\phi_{II} = \int_0^1 y \phi(y) dy \tag{17a}$$

$$I_0^* = \tfrac{1}{3} m l^3 \tag{17b}$$

Solving the system of Eq. (16) for $\Delta\dot{\theta}$ and $\Delta\dot{q}$ leads to

$$\Delta \dot{\theta} = A \Delta v$$

$$\Delta \dot{q} = B \Delta v \tag{18}$$

where

$$A = \frac{\tfrac{1}{2} m^* m l^2 - m^2 l \phi_{II}}{m^* I_0^* - (m \phi_{II})^2}$$

$$B = \frac{m I_0^* l - \tfrac{1}{2} m^2 l^2 \phi_{II}}{m^* I_0^* - (m \phi_{II})^2} \tag{19}$$

Similarly the kinetic energy (KE) for the whole beam is obtained by integrating Eq. (13) for the overall length and use of Eq. (17), that is

$$\mathrm{KE} = \tfrac{1}{2} I_0^* \dot{\theta}^2 + m \phi_{II} \dot{\theta} \dot{q} + \tfrac{1}{2} m^* \dot{q}^2 \tag{20}$$

If the rocking beam is subjected to a series of n random ground impulses, the incremental change in kinetic energy $\Delta \mathrm{KE}_n$ for the nth impulse is given by

$$\Delta \mathrm{KE}_n = \tfrac{1}{2} I_0^* (\dot{\theta}_n + \Delta \dot{\theta}_n)^2 - \tfrac{1}{2} I_0^* (\dot{\theta}_n)^2 + m \phi_{II} (\dot{\theta}_n + \Delta \dot{\theta}_n)(\dot{q}_n + \Delta \dot{q}_n) - m \phi_{II} \dot{\theta}_n \dot{q}_n + \tfrac{1}{2} m^* (\dot{q}_n + \Delta \dot{q}_n)^2 - \tfrac{1}{2} m^* (\dot{q}_n)^2 \tag{21}$$

For earthquake excitations, it can be assumed that the average Δv is equal to zero (Housner 1963). Consequently, substituting

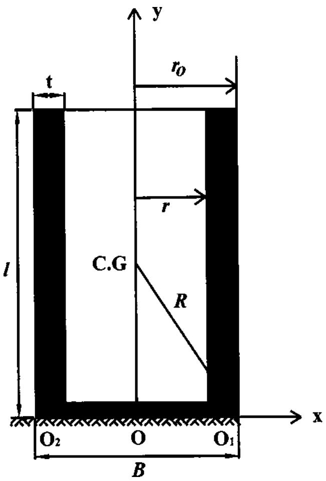

Fig. 3. Cross section of tower elevation

$\Delta\dot{\theta}$ and $\Delta\dot{q}$ from Eq. (18) into Eq. (21), performing the algebraic calculations, and letting the average Δv to be equal to zero, Eq. (21) results in

$$\Delta \mathrm{KE}_n = T_K \Delta v^2 \tag{22}$$

where

$$T_K = \tfrac{1}{2} I_0^* A^2 + mAB\phi_{II} + \tfrac{1}{2} m * B^2 \tag{23}$$

Eq. (22) expresses the energy change caused by an earthquake excitation.

As shown in Fig. 2 overturning occurs through rotation of the structure about one of the bottom corners. If the base of the structure is narrow, one can neglect the effect of the width B and consider that overturning occurs through rotation about the center O. The total potential energy change ΔPE, that is attributed to the height difference between the initial position $\theta = 0$ and the final position $\theta = \alpha$, shown in Fig. 2, is given by

$$\Delta \mathrm{PE} = \int_0^1 mg[y - y\cos a + u \sin a]dy \tag{24}$$

For small α, Eq. (24) can be simplified to yield

$$\Delta \mathrm{PE} = W\left[\tfrac{1}{4} l a^2 + \frac{\phi_I}{l} aq\right] \tag{25}$$

where W = total weight of the structure. The first term in Eq. (25) is related to rigid body motion, while the second term is related to structural flexibility. In Eq. (25), and for a slender structure, one may neglect the effect of the width B on the rigid body motion to obtain

$$\Delta \mathrm{PE} = W\left[\tfrac{1}{2} R a^2 + \frac{\phi_I}{l} aq\right] \tag{26}$$

where R is practically half the height of the structure and $\alpha \approx \tan\alpha = B/l$ [see Fig. 2(a)].

The strain energy attributed to flexibility is given by (Craig 1981)

Table 1. Critical Spectral Velocity (S_{v0}) for Tower without Foundation

		Rigid block			Flexible		Difference	
		S_{v0}(m/s)			S_{v0}(m/s)			
B (m)	l/B	Eq. (33)	Eq. (34)	Eq. (35)	Eq. (31), cantilever beam	Eq. (31), free beam	[Eq. (35)−Eq. (34)]/Eq. (35) (%)	[Eq. (35)−Eq. (31) cantilever beam]/Eq. (35) (%)
	5	3.43	3.96	4.00	2.43	2.62	0.98	39.36
	6	3.13	3.62	3.64	2.22	2.38	0.68	39.02
	7	2.90	3.35	3.36	2.06	2.20	0.50	38.66
	8	2.71	3.13	3.14	1.94	2.06	0.39	38.27
	9	2.56	2.95	2.96	1.84	1.94	0.31	37.85
12	10	2.43	2.80	2.81	1.76	1.84	0.25	37.40
	11	2.31	2.67	2.68	1.69	1.75	0.21	36.93
	12	2.21	2.56	2.56	1.63	1.68	0.17	36.43
	13	2.13	2.46	2.46	1.58	1.61	0.15	35.91
	14	2.05	2.37	2.37	1.53	1.55	0.13	35.36
	15	1.98	2.29	2.29	1.49	1.50	0.11	34.79
	5	2.43	2.80	2.83	1.71	1.85	0.98	39.62
	6	2.21	2.56	2.57	1.56	1.69	0.68	39.38
	7	2.05	2.37	2.38	1.45	1.56	0.50	39.11
	8	1.92	2.21	2.22	1.36	1.46	0.39	38.82
	9	1.81	2.09	2.09	1.29	1.37	0.31	38.51
6	10	1.72	1.98	1.99	1.23	1.30	0.25	38.18
	11	1.64	1.89	1.89	1.18	1.24	0.21	37.84
	12	1.57	1.81	1.81	1.13	1.19	0.17	37.47
	13	1.50	1.74	1.74	1.09	1.14	0.15	37.09
	14	1.45	1.67	1.68	1.06	1.10	0.13	36.69
	15	1.40	1.62	1.62	1.03	1.06	0.11	36.27

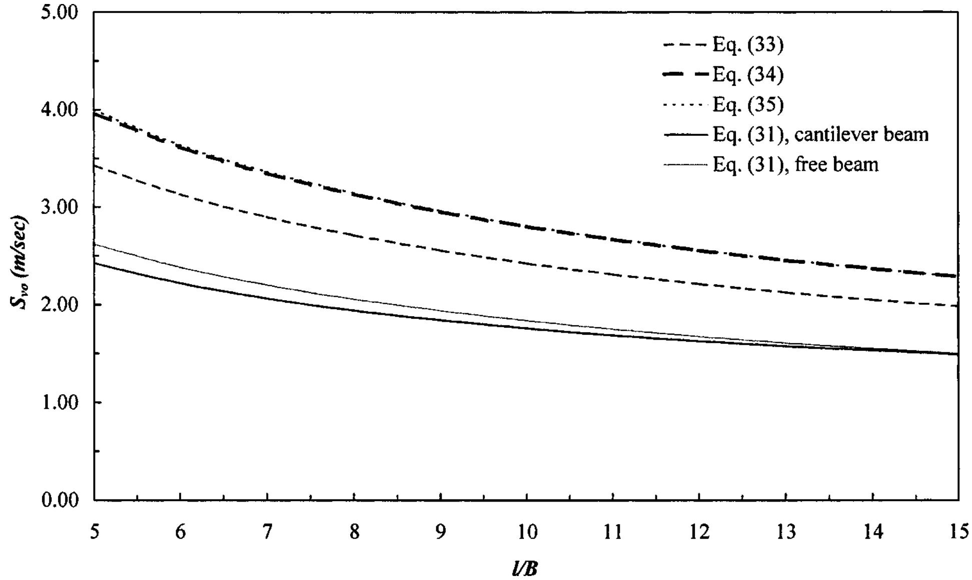

Fig. 4. Variation of critical spectral velocity S_{v0} with slenderness ratio l/B for tower without foundation (B=12 m)

$$U=K_e q^2 \tag{27}$$

where

$$K_e=\int_0^1 \tfrac{1}{2}EI\phi''(y)^2 dy \tag{28}$$

The condition for overturning at the end of n pulses is developed by equating the difference between the total potential energy change ΔPE and the strain energy U to the energy change caused by the earthquake excitation ΔKE_n, that is

$$W\left[\tfrac{1}{2}Ra^2+\frac{\phi_I}{l}aq\right]-K_e q^2=T_K(\Delta v)^2 \tag{29}$$

If the energy input is computed from the velocity response spectrum of the earthquake ground motion, Eq. (29) can be written in terms of Eq. (10) as

$$W\left[\tfrac{1}{2}Ra^2+\frac{\phi_I}{l}a\frac{m\phi_I}{m^*\omega}S_{v0}\right]-K_e\left(\frac{m\phi_I}{m^*\omega}\right)^2 S_{v0}^2=T_K S_{v0}^2 \tag{30}$$

where S_{v0}, the critical overturning velocity, denotes the spectral velocity that will cause overturning of the structure. With simple algebraic manipulations Eq. (30) takes the form

$$\left[T_K+K_e\left(\frac{m\phi_I}{m^*\omega}\right)^2\right]S_{v0}^2-W\frac{m\phi_I^2 a}{m^* l\omega}S_{v0}-\tfrac{1}{2}WRa^2=0 \tag{31}$$

Eq. (31) establishes an overturning criterion as a function of the critical overturning velocity S_{v0}, the inertial and the geometric parameters of a flexible system. For a given site and tower height, Eq. (31) can be used in a preliminary design to select the appropriate dimensions for the foundation in order to avoid overturning. Furthermore, the same equation could be used to address the inverse problem that is to estimate the ground velocity that caused

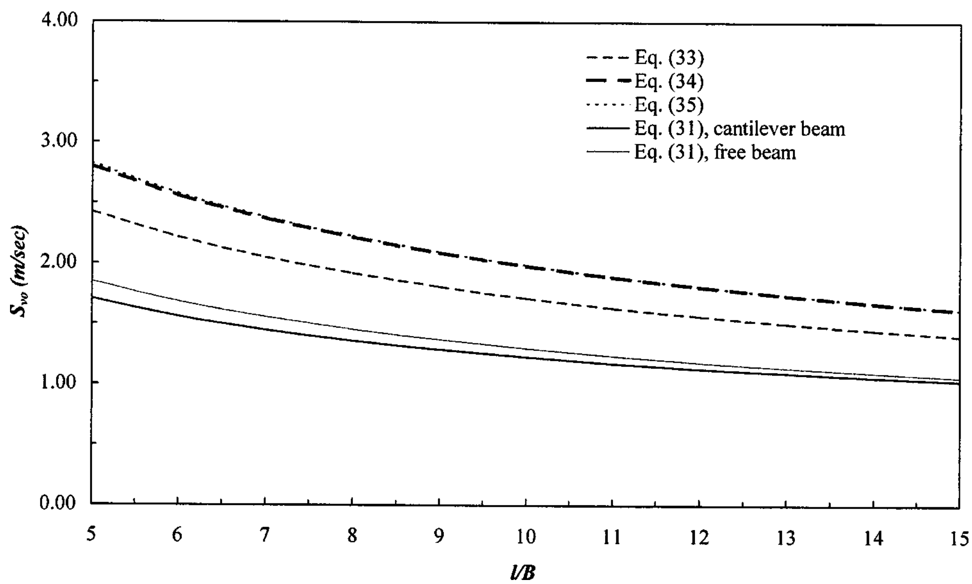

Fig. 5. Variation of critical spectral velocity S_{v0} with slenderness ratio l/B for tower without foundation (B=6 m)

the overturn of a slender structure, a procedure that has been the subject of several studies on the overturn of primarily rigid bodies (e.g. Apostolou et al. 2001).

If structural flexibility is neglected, Eq. (31) can be simplified to the following form that has been proposed by Housner (1963):

$$a = \frac{S_{v0}}{\sqrt{gR}}\sqrt{\frac{MR^2}{I_0^*}} \tag{32}$$

where M=total mass of the system.

The procedure to arrive at Eq. (32) involves elimination of the second and third terms in Eq. (31) and the last two terms on the right hand side of Eq. (23).

For slender structures, if we assume that MR^2/I_0^* has a value close to unity (Housner 1963), then Eq. (32) takes the form

$$a = \frac{S_{v0}}{\sqrt{gR}} \tag{33}$$

If MR^2/I_0 is set as equal to 3/4 (Ishiyama 1980), then Eq. (32) takes the form

$$a = \sqrt{\frac{3}{4gR}}S_{v0} \tag{34}$$

It should be noted that in both Eqs. (33) and (34), R is assumed to be equal to half the height of structure [see Fig. 2(a)]. A more accurate consideration would be to consider the moment of inertia with respect to the corner point O_1 and the effect of the width B. In this case α is evaluated from

$$a = \frac{S_{v0}}{\sqrt{gR}}\sqrt{\frac{MR^2}{I_{O_1}}} \tag{35}$$

Numerical Examples

The significance of either accounting for or ignoring flexibility in overturning criteria is investigated through parametric studies and comparisons. Parametric studies are performed for a tower with the geometry shown in Fig. 3. The tower foundation simply rests on the ground without any anchorage that could restrain uplift and overturn. The study also examines the relative significance of various geometric parameters for the tower simulated as a rigid body. In all studies the width of the structure B is assigned two different values: $B=2r_0=6$ m and $B=2r_0=12$ m. For the tower with $B=6$ m, the height l varies from 30 to 90–m in increments of 6 m, the inner radius is $r=2.4$ m, and the outside radius is $r_0=3$ m. For the tower with $B=12$ m, the height l varies from 60 m 180 m in increments of 12 m, the inner radius is $r=5.4$ m, and the outside radius is $r_0=6$ m. The tower is made of concrete with a Young's modulus $E=31$ GPa and a unit weight of 24.35 kN/m^3.

For the case of a rigid tower the critical overturning velocity S_{v0} is evaluated for varying B and l/B as expressed by Eqs. (33)–(35) and the results are presented in Table 1. The difference between the criteria expressed by Eqs. (33)–(35) is that the first

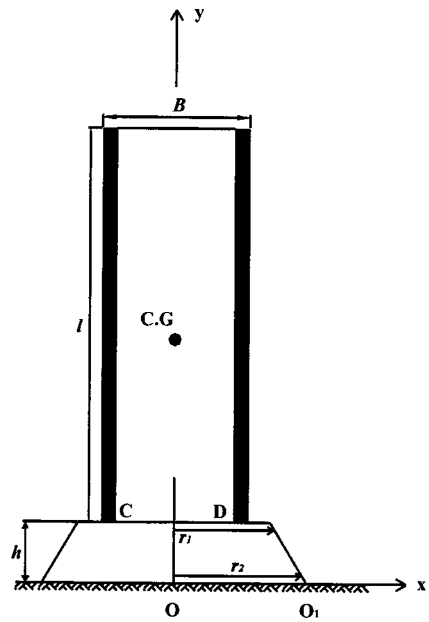

Fig. 6. Tower with tapered foundation

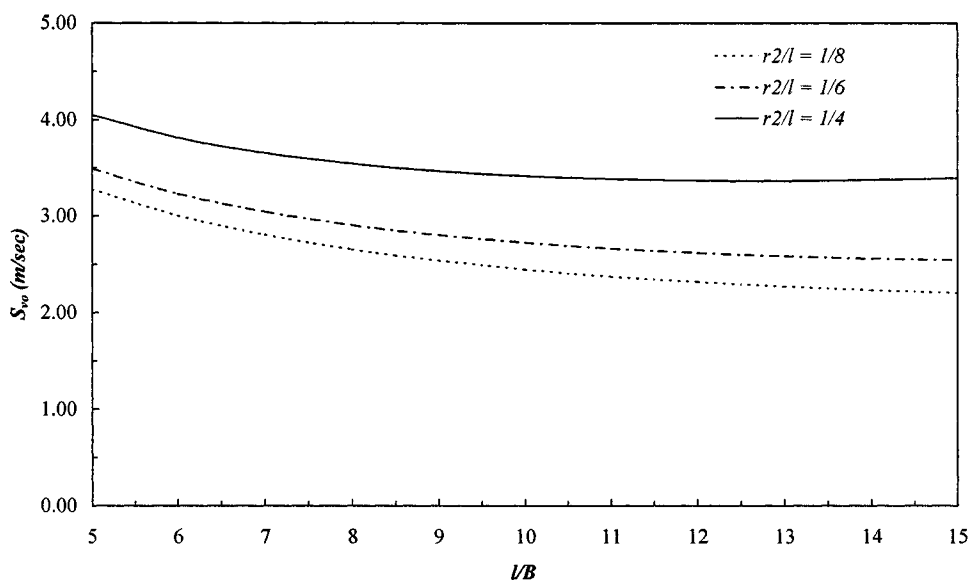

Fig. 7. Variation of critical spectral velocity S_{v0} with slenderness ratio l/B for different values of r_2/l (B=12 m)

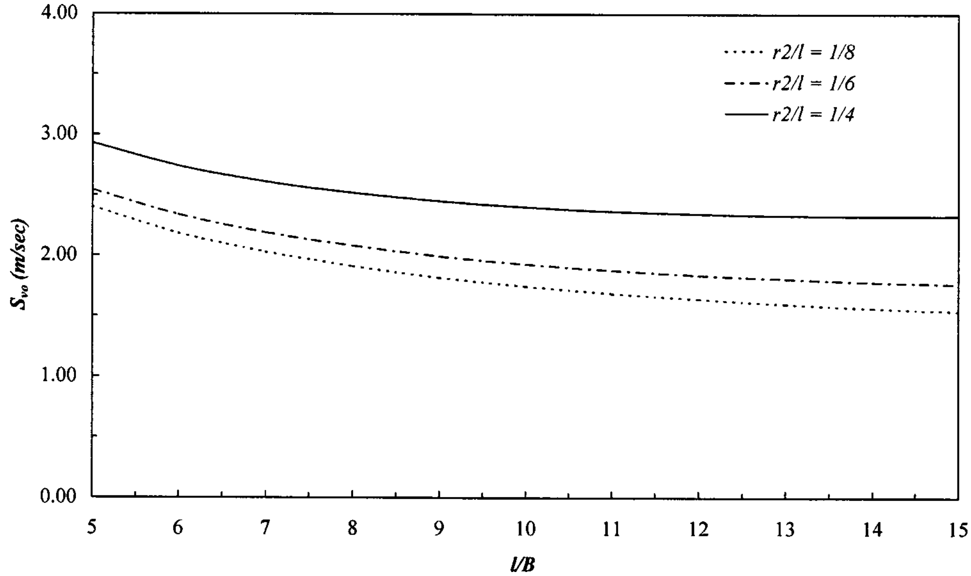

Fig. 8. Variation of critical spectral velocity S_{v0} with slenderness ratio l/B for different values of r_2/l (B=6 m)

two equations are based on the evaluation of I_0^* with respect to the base center of the tower, while the latter uses I_{O1} which refers to one of the bottom edges of the foundation. The information in Table 1 is also presented in Figs. 4 and 5 for B=12 m and B =6 m, respectively. It is observed that Housner's calculation of α given by Eq. (33) is more conservative than Ishiyama's, expressed by Eq. (34). The results in column 8 of Table 1 clearly demonstrate that the results of Eq. (34) are almost identical to the ones obtained by Eq. (35). Therefore use of the moment of inertia with respect to the base center is reasonable for slender towers.

The overturning stability for a tower with the geometry shown in Fig. 3 but including flexibility is evaluated using Eq. (31). Figs. 4 and 5 also show the critical spectral velocity S_{v0} for B=12 and 6 m, respectively. Two different cases in terms of boundary conditions, i.e., a cantilever beam and a free beam, are examined. It is observed that, although for lower values of l/B a free beam results in a higher critical overturning velocity S_{v0}, both cases converge to practically the same solution for high values of the slenderness ratio. Table 1 demonstrates the difference of either including or ignoring tower flexibility in overturning stability. As shown in column 9 of Table 1 the critical spectralvelocity of the flexible tower is less than that of the tower modeled as a rigid block with the same geometric dimensions. This implies that estimating S_{v0} based on the idealization of the tower as a rigid body is not conservative. For most cases the reduced spectral velocity of the flexible idealization is about 37% of the rigid model.

The typical case of a hollow cylindrical tower supported by a rigid circular slopped foundation is also investigated (see Fig. 6). It should be noted that the tower flexibility is taken into account in all parametric studies in order to assess the effect of the foundation on the overturning stability of the superstructure because such a model is considered as a more realistic approximation of the actual structural system compared to the rigid body idealization. The presence of the foundation complicates the derivation of the overturning criterion. Nevertheless, Eq. (31) can still be used as an overturning criterion. However, use of Eq. (31) requires evaluation of both the fundamental frequency of the tower with the foundation. Therefore, Eq. (17*b*) is modified in order to account for the moment of inertia of the whole structure with respect to the edge point O_1 as shown in Fig. 6. Furthermore, the connection of the foundation to the tower is assumed to be monolithic and, therefore, the normal shape corresponding to the first mode of a cantilever beam, that is Eq. (3), is used for the overturning criterion.

Parametric studies are performed for a hollow cylindrical tower bonded to a slopped foundation (Fig. 6) in order to examine

Table 2. Critical Spectral Velocity (S_{v0}) for Tower Supported by Foundation with Variable Diameter

		Flexible		
		S_{v0} (m/s)		
B(m)	l/B	r_2/l=1/8	r_2/l=1/6	r_2/l=1/4
	5	3.28	3.49	4.05
	6	3.00	3.23	3.82
	7	2.81	3.04	3.66
	8	2.66	2.91	3.54
	9	2.54	2.80	3.47
12	10	2.45	2.72	3.42
	11	2.37	2.66	3.38
	12	2.32	2.62	3.37
	13	2.27	2.58	3.36
	14	2.23	2.56	3.37
	15	2.20	2.54	3.39
	5	2.40	2.55	2.93
	6	2.18	2.34	2.74
	7	2.03	2.19	2.61
	8	1.91	2.08	2.52
	9	1.82	1.99	2.45
6	10	1.74	1.93	2.40
	11	1.69	1.88	2.37
	12	1.64	1.84	2.34
	13	1.60	1.81	2.33
	14	1.57	1.78	2.33
	15	1.54	1.76	2.33

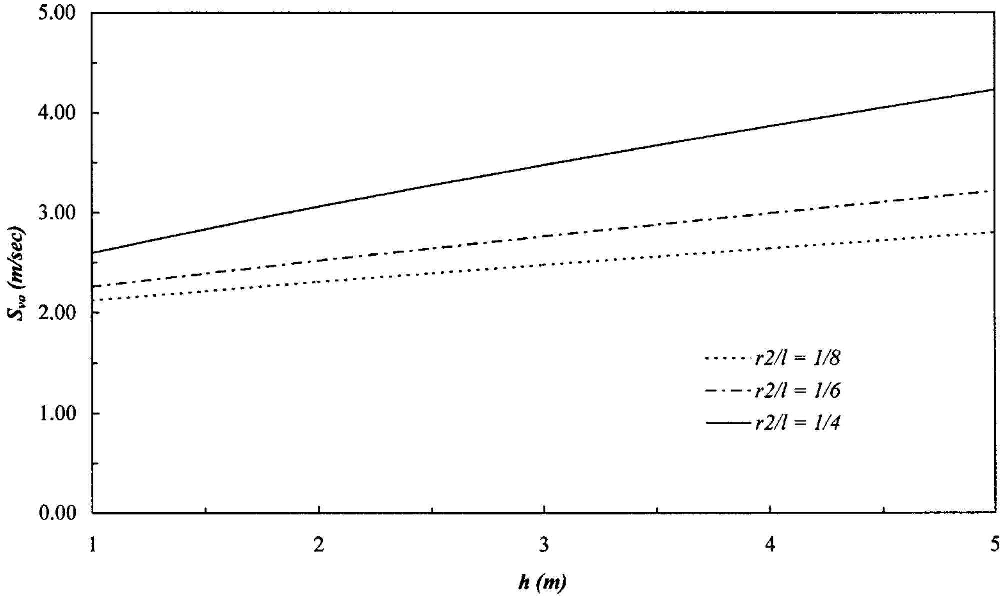

Fig. 9. Variation of critical spectral velocity S_{v0} with foundation height h for different values of r_2/l ($B=12$ m)

the effect of slenderness on overturning stability. The width of the structure is assigned two different values: $B=2r_0=6$ m and $B=2r_0=12$ m. Figs. 7 and 8 illustrate the critical spectral velocity for $B=12$ and 6 m, respectively, as a function of the slenderness ratio l/B, while Table 2 lists the resulting S_{v0} for three representative values of the r_2/l ratio (Pinfold 1975). For the tower with $B=6$ m, the foundation height h is 3 m, the top radius r_1 is 3.5 m, and the bottom radius r_2 varies from 3.75 to 22.5 m. For the tower with $B=12$ m, the foundation height h is 3 m, the top radius r_1 is 6.5 m, and the bottom radius r_2 varies from 7.5 to 45 m. Both Figs. 7 and 8 demonstrate the same trend, i.e., the critical spectral velocity decreases with increasing slenderness ratio in an almost linear variation. Furthermore, flexible towers supported by a narrow foundation are more vulnerable to overturn due to ground motion compared to towers on a wider foundation.

In Figs. 9 and 10, the critical spectral velocity S_{v0} is plotted against the foundation height h for $B=12$ and 6 m, respectively. It should be noted that for the tower with $B=6$ m, the height l is 60 m, based on a slenderness ratio equal to ten, the top radius r_1 is 3.5 m, and the bottom radius r_2 varies from 7.5 to 15 m. For the tower with $B=12$ m, the height l is 120 m ($l/B=10$), the top radius r_1 is 6.5 m, and the bottom radius r_2 varies from 15 to 30 m. It is observed that S_{v0} increases with increasing foundation height. In both figures the increase is more apparent for higher values of r_2/l. As it can be seen in Table 3, for $B=12$ m, the critical velocity increases from 2.60 to 4.23 m/s for $r_2/l=1/4$, which is approximately 63%, while it only increases about 32% for $r_2/l=1/8$. Similarly, for $B=6$ m, the increase of the critical spectral velocity is 68% for $r_2/l=1/4$ and just 37% for $r_2/l=1/8$. It is also worth noting that the relationship between the

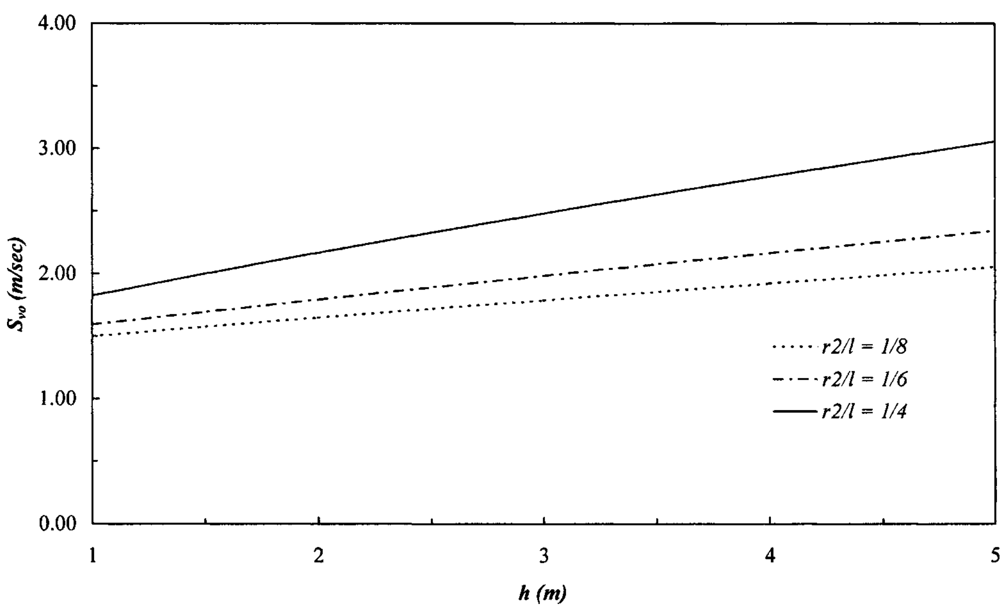

Fig. 10. Variation of critical spectral velocity S_{v0} with foundation height h for different values of r_2/l ($B=6$ m)

Table 3. Critical Spectral Velocity (S_{v0}) for Tower Supported by Foundation with Variable Height

B (m)	h (m)	Flexible S_{v0} (m/s) $r_2/l=1/8$	$r_2/l=1/6$	$r_2/l=1/4$
12	1	2.12	2.26	2.60
	2	2.31	2.52	3.06
	3	2.48	2.76	3.48
	4	2.64	2.99	3.86
	5	2.80	3.21	4.23
6	1	1.50	1.59	1.82
	2	1.65	1.79	2.17
	3	1.79	1.98	2.48
	4	1.92	2.17	2.78
	5	2.05	2.34	3.06

critical overturning velocity and the foundation height is almost linear for all the r_2/l ratios examined.

Conclusions

Criteria for the overturning stability of slender structures, such as chimneys and towers, based on a single mode domination of the dynamic characteristics of the structure are developed. The overturning criteria relate the structural stiffness and inertia parameters to the spectral pseudovelocity of the ground motion.

Numerical examples, parametric studies, and comparisons illustrate the significance of either accounting for or ignoring flexibility in overturning criteria. The parametric studies lead to the following conclusions: flexibility reduces the magnitude of spectral pseudovelocity that causes overturn. This implies that estimating S_{v0} based on the idealization of the tower as a rigid body is unconservative. The critical spectral velocity decreases for increasing slenderness ratio. Keeping the slenderness ratio constant, the critical spectral velocity causing overturning decreases for decreasing geometric size of the tower. Parametric studies also show that the presence of even a very narrow foundation has a beneficial effect on the overturning stability of a flexible tower. Furthermore, the vulnerability of the tower decreases by increasing either the foundation diameter or the foundation height.

The procedure could also be useful to address the inverse problem that is to estimate the ground velocity that caused the overturn of a slender structure.

Notation

The following symbols are used in this paper:

B = width of structure;
c = viscous damping;
E = Young's modulus;
g = acceleration of gravity (9.806 m/s^2);
I = second moment of area;
I_0^* = moment of inertia with respect to base center of structure;
I_{O1} = moment of inertia with respect to corner point of structure;
$\overline{KE}$ = kinetic energy per unit length;
KE = kinetic energy for whole beam;
l = beam length;
M = total mass of system;
M_b = overturning base moment;
m = mass per unit length of beam;
$q(t)$ = generalized coordinate;
R = distance between center of gravity and corner point of structure;
r = inner radius of hollow cylindrical tower;
r_0 = outside radius of hollow cylindrical tower;
r_1 = top radius of slopped foundation;
r_2 = bottom radius of slopped foundation;
S_v = pseudovelocity spectral response;
S_{v0} = critical overturning velocity;
U = strain energy attributed to flexibility;
$u(y,t)$ = relative displacement perpendicular to beam axis;
$\ddot{u}_g$ = ground excitation;
V_b = resistant base shear force;
W = total weight of structure;
α = rotational angle of structure when overturn occurs;
ΔKE_n = incremental change in kinetic energy for *n*th impulse;
ΔPE = total potential energy change;
Δt = step changes in time;
$\pm\Delta v$ = discrete step changes in ground velocity;
θ = rotational angle;
ξ = damping ratio;
$\phi(y)$ = normal mode shape; and
ω = natural frequency of the system.

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