# ESTIMATION OF INPUT SEISMIC ENERGY BY MEANS OF A NEW DEFINITION OF STRONG MOTION DURATION

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**ABSTRACT:**

This study presents a new method to estimate the earthquake input energy for medium to long period structures. The method is based on a new definition of the strong motion duration, combining the well known bracketed and significant duration definitions. The proposed duration presents high correlation with the duration of intense energy release. Two normalized parameters are introduced permitting a good approximation of the input energy by taking into account an effective value of the ground velocity and an equivalent number of excitation cycles depending on the frequency content of the ground velocity time history. The proposed parameters present a large correlation coefficient and the resulting error is minimal in comparison with existing methods that estimate the earthquake input energy.

**KEYWORDS:** seismic input energy, strong-motion duration, cycles of seismic loading

## 1. INTRODUCTION

Current design practices associated with performance criteria are based on the estimation of the deformation demand that must be satisfied during an earthquake excitation and which is usually addressed by the supply of adequate ductility. The availability of sufficient deformation capability is indirectly taken into account using a strength reduction factor that depends on the structural system. A new energy based approach considering the cumulative effects of the seismic load through a combination of response and energy absorption parameters is introduced to account for both the effects of duration and hysteretic behavior directly. A basic parameter for the implementation of energy based concepts is the estimation of the input energy that is considered to be a reliable indicator of ground motion severity.

Uang and Bertero (1990) proposed two different approaches to estimate the input energy, based on either the absolute or the relative equation of motion. Bruneau and Wang (1996), as well as Chopra (2006), suggested the definition of relative input energy, as more consistent. The relative input energy is given by the following expression:

$$\frac{m(\dot{v})^2}{2} + \int c\dot{v}dv + \int f_s dv = -\int m\ddot{v}_g dv \Rightarrow E'_K + E_\xi + E_a = E'_I \quad (1.1)$$

where, $v$ and $v_g$ are relative and ground displacement, respectively, and $E'_K$, $E_\xi$, $E_a$ and $E'_I$ are the "relative" kinetic energy, damping energy, absorbed energy and "relative" input energy, respectively.

Results by previous investigators, as Akiyama (1985) and Zahrah and Hall (1984) have indicated that the maximum input energy per unit mass has a relatively stable value in the region of the predominant period of the ground motion that is not greatly affected by the structural parameters as damping structural strength and hysteresis response model. The most significant modification is that, as the target ductility increases, a shift of the maximum values towards shorter periods is observed, attributed to the change of the effective period for pronounced inelastic behaviour. The input energy is a quite stable parameter regarding the effects of structural parameters on the structural response. In contrast, the influence of the characteristics of the ground motion on the input energy is quite significant. All of the relevant parameters such as amplitude, duration and frequency content seem to substantially affect the input energy.

A first estimation of the input energy per unit mass, based on the maximum kinetic energy, has been presented by Housner (1956) for both elastic and inelastic behaviour

$$E_I = \frac{1}{2} PSV^2 \tag{1.2}$$

where PSV is the spectral pseudovelocity.

A different formula has been proposed by Fajfar *et al.* (1992) in the form

$$\frac{E_I}{m} = 0.85 \cdot \frac{PGV}{PGA} \cdot \int_0^{t_r} a^2 dt \tag{1.3}$$

where the predominant period of the earthquake record is proportional to the ratio between the peak ground velocity *PGV* and the peak ground acceleration *PGA*. Fajfar *et al.* (1989) have proposed a different formula taking into account the peak ground velocity and the significant duration $t_d$ as defined by Trifunac and Brady (1975) according to the following expression

$$\frac{E_I}{m} = 2.2 \cdot t_d^{0.5} \cdot PGV^2 \tag{1.4}$$

The same authors state that the dispersion of results proves that their formula is not suitable for long duration ground motions with a short predominant period or for motions with a large peak connected with a long pulse. In the former case the results are too small and in the later too large.

In this study a sample of earthquake records was used in order to estimate the correlation between input energy and ground motion parameters. As an appropriate index, the elastic input energy for 5% damping corresponding to the predominant period $T_{d\text{-}p}$ of the displacement spectrum is selected. The period $T_{d\text{-}p}$ is closely associated to the period $T_p$ of the directivity pulses contained in the velocity time-history, according to Mavroeidis *et al.* (2004). The period $T_p$ defines the region of increased ductility demand and is close to the transition zone between the constant velocity and displacement regions of the response spectrum. Consequently, the elastic input energy $EIT_{d\text{-}p}$ can be considered as an index characterizing the earthquake energy input for middle and long period structures.

## 2. DEFINITIONS AND DESCRIPTION OF METHODOLOGY

### *2.1. New Definition of Duration*

In this work emphasis is placed on the medium and long period region of the elastic input energy spectrum, a region dominated by the amplitude and frequency content of the ground velocity. Furthermore, the intensity and the energy content of near source strong ground motions is closely related to the amplitude and number of ground velocity pulses.

Since the ground velocity is associated with the energy released at the recording site, it is proposed that the significant duration of the ground motion, associated with intense energy release, should be related to the steep gradient of the time integral of the absolute velocity, instead of the Arias integral (Arias, 1970). For this reason, the time integral of absolute ground velocity is introduced, in analogy with the already established definition of the cumulative absolute velocity CAV (EPRI, 1991). The new index is defined as the cumulative absolute displacement, CAD, that is

$$CAD = \int_0^{tr} \left| v_g \right| dt \tag{2.1}$$

where $t_r$ is the total duration of the acceleration trace.

The introduction of CAD, also allows a combination of the definitions of the significant and bracketed duration, since the gradient of the time integral is equal to the absolute velocity. For each ground motion, a threshold relative to a percentage of the maximum ground velocity can be defined, so that the subsequent bracketed duration coincides with the significant duration encompassing the steep gradient of the absolute velocity integral. In this study a threshold of approximately 30% of the maximum ground velocity is used in order to define a bracketed duration. The bracketed duration is found to be especially well correlated with the steep gradient of the time integral of the absolute ground velocity. The steep gradient of the CAD integral coincides with the largest ground velocity and consequently with the occurrence of the intense earthquake energy release, so that it can be termed as the significant duration of the ground motion. This duration is characterized as bracketed-significant $t_{bs}$, since it combines both definitions. Once the duration $t_{bs}$ is defined, an effective velocity index $V_{mean}$ is estimated as the average gradient of the steep portion of the CAD integral as follows:

$$V_{mean} = \frac{\int_{t_1}^{t_2} \left| v_g(t) \right| dt}{t_{bs}} \tag{2.2}$$

where $t_1$ and $t_2$ are the limits of the bracketed-significant duration $t_{bs}$.

### *2.2. Energy Input and Time History Correlation*

The present study adopts Housner's (1975) suggestion to use two parameters to define the ground motion severity in order to deaggregate the amplitude and duration effects. In order to establish a relationship between ground motion characteristics and the associated energy input, as expressed by $EI_{Td\text{-}p}$, two normalized parameters $P_1$ and $P_2$ are defined. The parameters $P_1$ and $P_2$ are correlated in accordance with the well known observation that the level of input energy is associated with the number of loading cycles through which the seismic energy is distributed.

The parameter $P_1$ is associated with the equivalent number of loading cycles contained in the time-history of ground velocity. According to Rodriguez-Marek (2000) the mean period of the individual velocity pulses of the ground velocity is very well correlated with the period $T_p$. Subsequently, in accordance with the indirect counting method, the equivalent number of cycles $P_1$ can be defined as the ratio

$$P_1 = \frac{t_{bs}}{T_{d-p}} \tag{2.3}$$

The relationship between the duration and the energy input of the ground motion depends on the effective value of the strong ground motion amplitude. Consequently, a second normalized parameter $P_1$, relating the elastic input energy $EI_{Td-p}$ to the effective ground velocity amplitude $V_{mean}$, is defined as:

$$P_2 = \frac{EI_{T_{d-p}}}{V_{mean}^2} \tag{2.4}$$

This ratio presents a spectral amplification referring to the effective instead of the peak value of the ground velocity time history.

### *2.3. Description of Methodology*

The following methodology has been employed in order to analyze selected earthquake records and estimate the normalized parameters $P_1$ and $P_2$ correlating time history and earthquake input energy quantities: a) first, for each record, the displacement and elastic input energy spectra for 5% damping and for a period range between 0.02 and 10.0 sec are constructed. Spectral values up to 10.0 sec are included, since large magnitude events produce near source records with predominant periods over 5.0 sec, b) the CAD integral is calculated, c) based on the CAD integral and the 5% elastic input energy spectra, the following procedure that consists of five steps is applied:

i) An optimum threshold of 30% of the peak ground velocity is considered in order to evaluate the proposed bracketed-significant duration $t_{bs}$. The duration $t_{bs}$ evaluated encompasses the portion of the CAD.
ii) Once the related duration is defined, the effective velocity $V_{mean}$ is calculated according to Eqn. 2.2.
iii) From the elastic input energy spectrum the spectral value $EI_{Td-p}$ corresponding to $T_{d-p}$ is evaluated. The parameter $P_2$ is defined according to Eqn. 2.4.
iv) The number of equivalent cycles $P_1$ is calculated according to Eqn. 2.3.
v) The sample of $P_1$ and $P_2$ values is used to draw a fitting curve that presents the relation between the energy input in the medium-long period region and ground velocity time-history indices.

In contrast with well known methodologies that utilize one or two of the time-history quantities, the proposed normalized parameters $P_1$ and $P_2$ combine ground motion duration, effective amplitude and frequency content information with structural response.

## 3. NUMERICAL RESULTS AND DISCUSSION

The earthquake records used in this study have been selected from the COSMOS and PEER databases. The records should be related to well known events from all over the world, with different levels of magnitude and short, medium and long significant durations. The sites of the recording stations present different soil conditions and source distances. Different directivity effects were taken into account.

The data sample includes well-known earthquakes, such as the Northridge (USA, 1994), the Kobe (Japan, 1995) and the Chi-Chi (Taiwan, 1999) events.

For near source records $t_{bs}$ is very close to the duration of the strong velocity pulses. Figures 1 and 2 show representative examples of the $t_{bs}$ duration for the E04-230 record of Imperial Valley (USA, 1979) and the ERZ-000 of the Erzincan (Turkey, 1992) events. For the presented near field records the bracketed-significant duration $t_{bs}$ coincides with the interval of intense energy release.

Figure 3.1 presents the sample and the fitting curve of the $P_1$ and $P_2$ values. It is observed that the relationship is almost linear since the fitting regression line has a coefficient of determination 0.95. This behavior indicates that the input energy increases in accordance with the number of loading cycles. As shown in figure 4.1, the maximum residuals between the sample data and the regression equation rarely exceed 100% of the predicted value which is considered as a good fit taking into consideration the uncertainty characterizing input energy values. The least squares fitting curve is given by the following expression:

$$P_2 = 21.351 \cdot P_1 - 16.202 \qquad (3.1)$$

The values of parameter $P_2$ are small when associated with near source records characterized by forward directivity phenomena with up to two or three strong velocity cycles in the strong motion part of the record. Greater $P_2$ values are associated with records presenting backward directivity effects, characterized by a large number of significant velocity pulses.

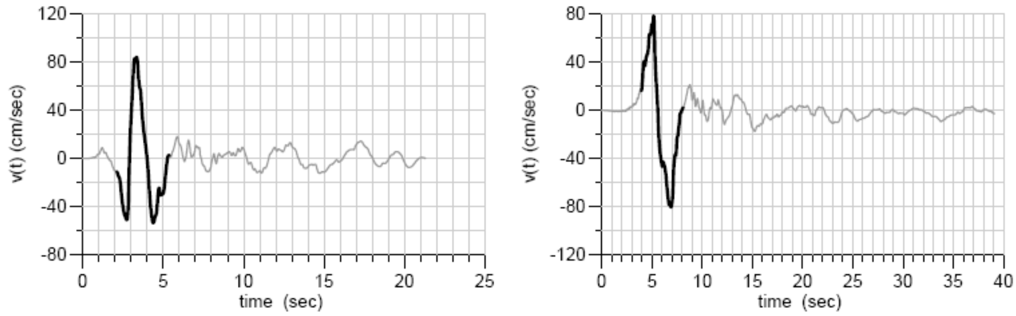



Figure 1 Bracketed-significant duration $t_{bs}$ portion (black trace) for the ERZ-000 velocity time-history of the Erzincan (1992) (left) and the E04-230 velocity time-history of the Imperial Valley (1979) events (right)

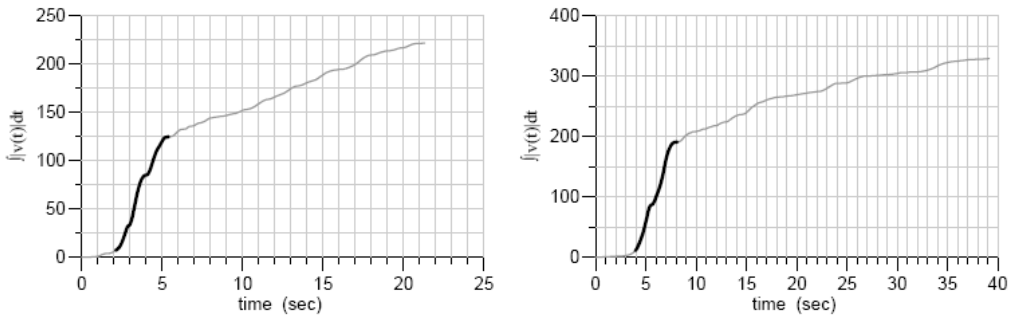
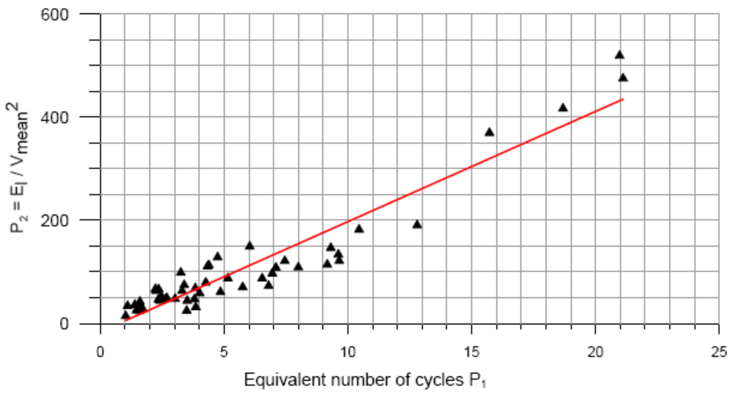



Figure 2 CAD integral (grey trace) with the corresponding bracketed-significant duration $t_{bs}$ portion (black trace) for the ERZ-000 component of the Erzincan (1992) (left) and the E04-230 component of the Imperial Valley, (1979) earthquakes (right)

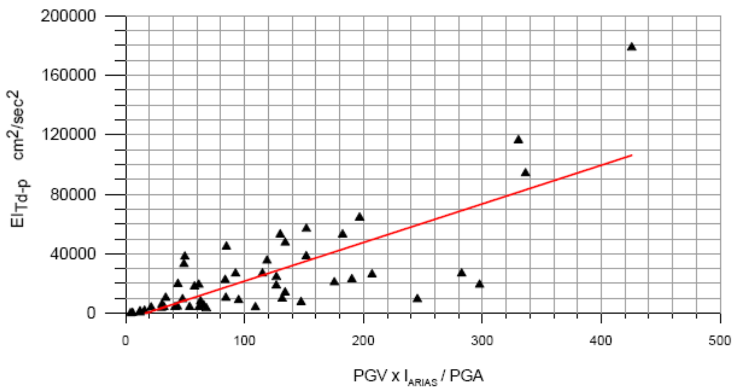
In order to evaluate the efficiency of the proposed parameters, the correlation between input energy and well established indices, as those presented by Fajfar *et al.* (1989) in Eqn. 1.3 and Eqn. 1.4 is examined. Figure 3.2 depicts the $EI_{Td\text{-}p}$ variation in terms of the index presented in Eqn. 1.3. Figure 3.3 depicts the $EI_{Td\text{-}p}$ variation in terms of the index introduced by Fajfar *et al.* (1989) in Eqn. 1.4. The least square fits are drawn with correlation coefficients of about 76% and 86% respectively. The related residuals, shown in figures 4.2 and 4.3, present values greater than 800% and 3500% of the predicted values, especially for records with backward directivity effects. Furthermore, the residuals of the index presented in Eqn. 2.4 appear to be related to the number of equivalent cycles $P_1$. The residuals regarding the $P_1$ and $P_2$ values are not related to the number of equivalent cycles, as presented in Figure 4.1.

## 3. CONCLUSIONS

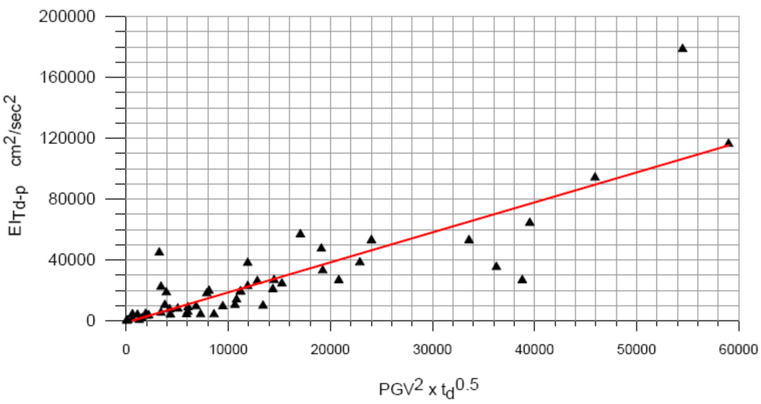
This study introduces a new method to estimate of the elastic input energy in the region of medium to large period structures. The method is based on a new definition of bracketed-significant duration. Instead of the squared acceleration and the associated Arias integral (Arias, 1970), the time integral of the absolute velocity is adopted as the pertinent parameter. Since the gradient of the integral is equal to the absolute velocity, use of a percentage of the maximum absolute velocity as a threshold defines a bracketed duration containing the significant part of the ground motion. The bracketed-significant duration encompasses the steep portion of the absolute velocity integral, expressed as cumulative absolute displacement CAD in analogy with the well known cumulative absolute velocity CAV (EPRI, 1991) index.




**(3.1)**




**(3.2)**

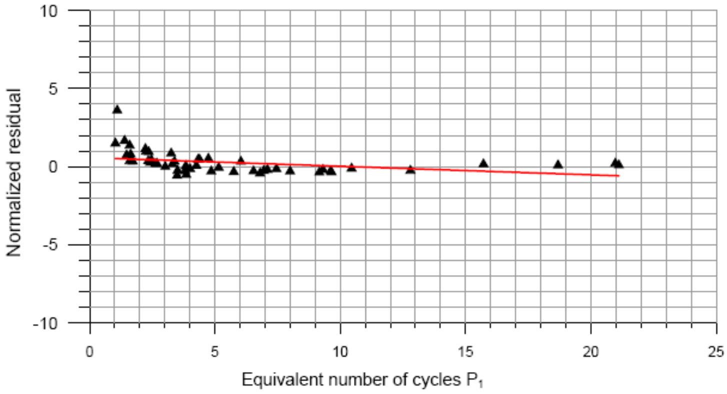



**(3.3)**

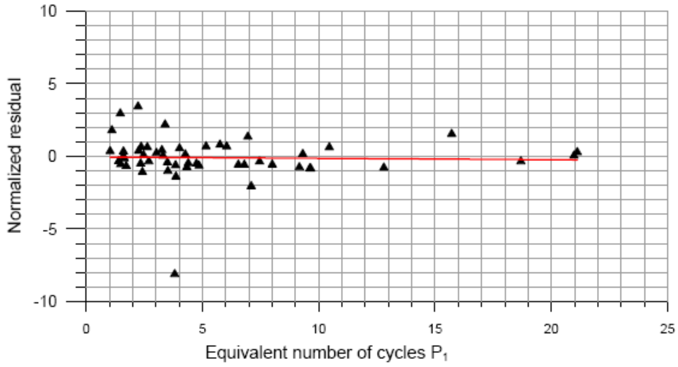
Figure 3 Correlation between: **(3.1)** the amplitude parameter $P_2$ and the equivalent number of cycles $P_1$, **(3.2)** the input energy with the index proposed in Eqn. 1.3, **(3.3)** the input energy with the index proposed in Eqn. 1.4

The computation of the elastic input energy is based on the mean gradient of the steep portion of the absolute velocity time integral and the number of the equivalent loading cycles. From the sample of 54 earthquake records the input energy spectra for 5% of the critical damping are computed. It should be noted that the bracketed-significant duration is found to coincide with the time interval during which the most intense energy absorption is observed.

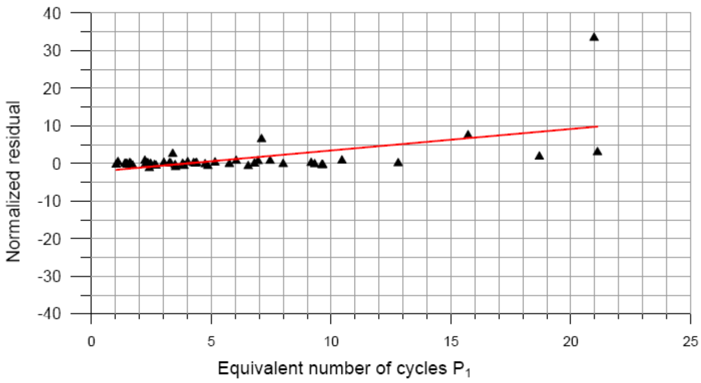
An index associated with the response of medium and longer period structures is defined as the maximum elastic input energy $EI_{Td\text{-}p}$ for 5% damping at the period $T_{d\text{-}p}$ that is closely related to the period of the ground velocity directivity pulses.




**(4.1)**




**(4.2)**




**(4.3)**

Figure 4 Correlation between the equivalent number of cycles $P_1$ and the normalized residuals: **(4.1)** of figure (3.1), **(4.2)** of figure (3.2), **(4.3)** of figure (3.3)

Two normalized parameters, $P_1$ and $P_2$, are introduced. The parameters $P_1$ and $P_2$ provide a good approximation of the elastic input energy in the medium to long period range with the aid of the indices $t_{bs,}$ CAD and $T_{d-p}$ that are associated with the ground velocity time-histories.

The correlation between $EI_{Td-p}$ and the indices proposed by Fajfar *et al.* (1989, 1992) are evaluated and relating fits are drawn. The correlation coefficients are found to be quite smaller than those between the newly proposed parameters $P_1$ and $P_2$. The related residuals present larger values and in one case are dependent on the number of equivalent velocity cycles.

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