NATIONAL TECHNICAL UNIVERSITY OF ATHENS
LABORATORY OF EARTHQUAKE ENGINEERING

Design of Earthquake-Resistant Structures

Basic principles

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Basic considerations

- Design earthquake: small probability of occurrence during the life of the structure.

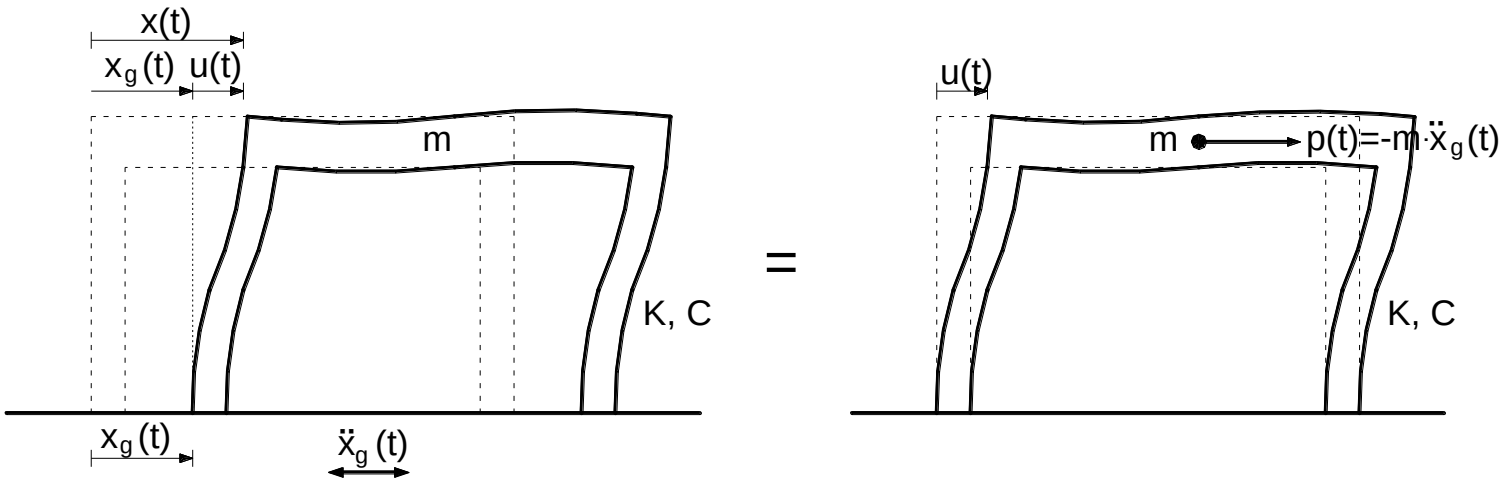
  It wouldn't be wise to design the structure to sustain this ground motion without any damage at all.

- Flexural yielding: the stiffness is temporarily reduced but the structure regains its strength and stiffness when unloading starts.

- Economical approach:

  - Allow the structure to exceed its elastic limit during the design earthquake.
  - Control the extend of the damage.
  - Exclude unwanted types of damage (e.g. brittle failure).
  - Repair any damage after the earthquake.

Seismic response of SDOF systems



Seismic forces (d' Alembert): $p(t) = -m \cdot \ddot{x}_g(t)$

Restoring force: $f_s = \sum V_i = \left(\sum K_i\right) \cdot u$

Damping force: $f_d(t) = C \cdot \dot{u}(t)$

Equation of motion: $p - f_s - f_d = m\ddot{u}$

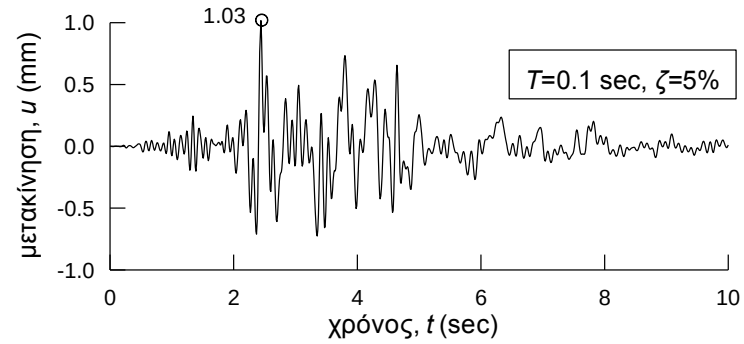
$$\boxed{m\ddot{u} + C\dot{u} + Ku = -m\ddot{x}_g}$$

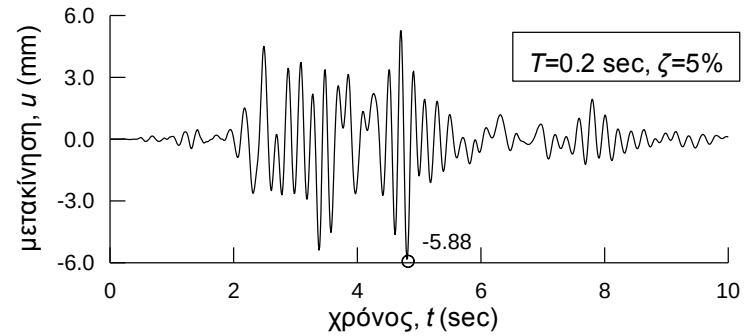
Seismic response of SDOF systems

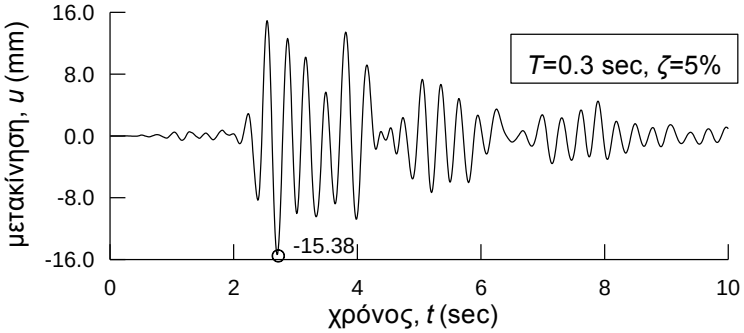
Equation of motion: $$\boxed{\ddot{u} + 2\zeta\omega\dot{u} + \omega^2 u = -\ddot{x}_g}$$

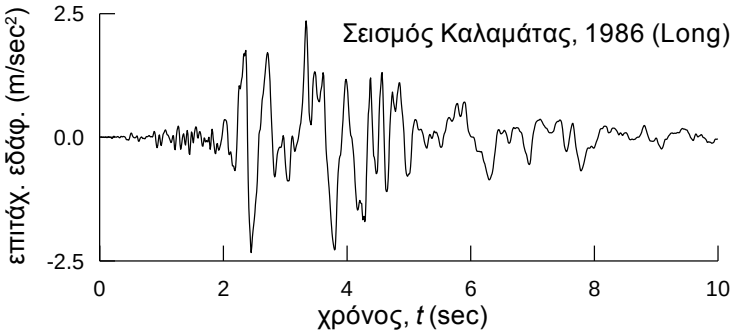
- Eigenfrequency: $$\omega = \sqrt{\frac{K}{m}}$$
- Eigenperiod: $$T = 2\pi\sqrt{\frac{m}{K}}$$
- Damping coefficient: $$\zeta = \frac{C}{2\sqrt{mK}}$$
- Duhamel's integral: $$u(t) = \frac{1}{\omega_d}\int_0^t \ddot{x}_g(\tau)\cdot e^{-\zeta\omega(t-\tau)}\cdot \sin\left[\omega_d(t-\tau)\right]\cdot d\tau$$

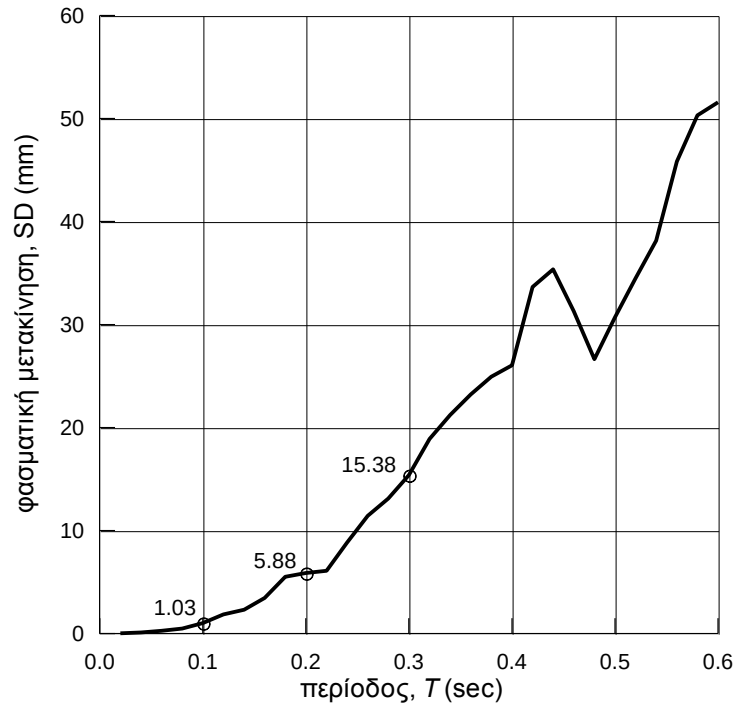
Response spectrum







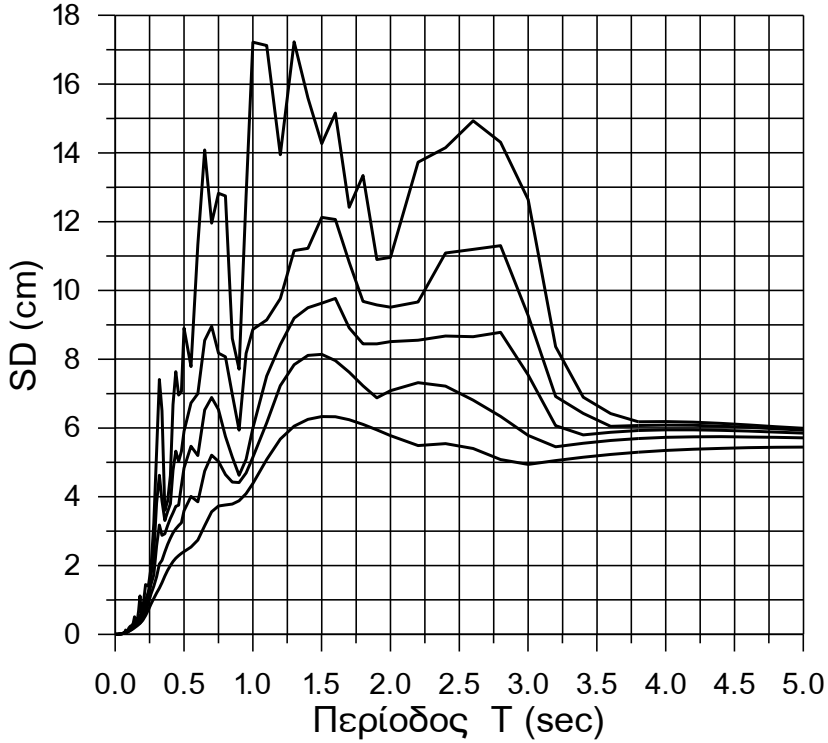


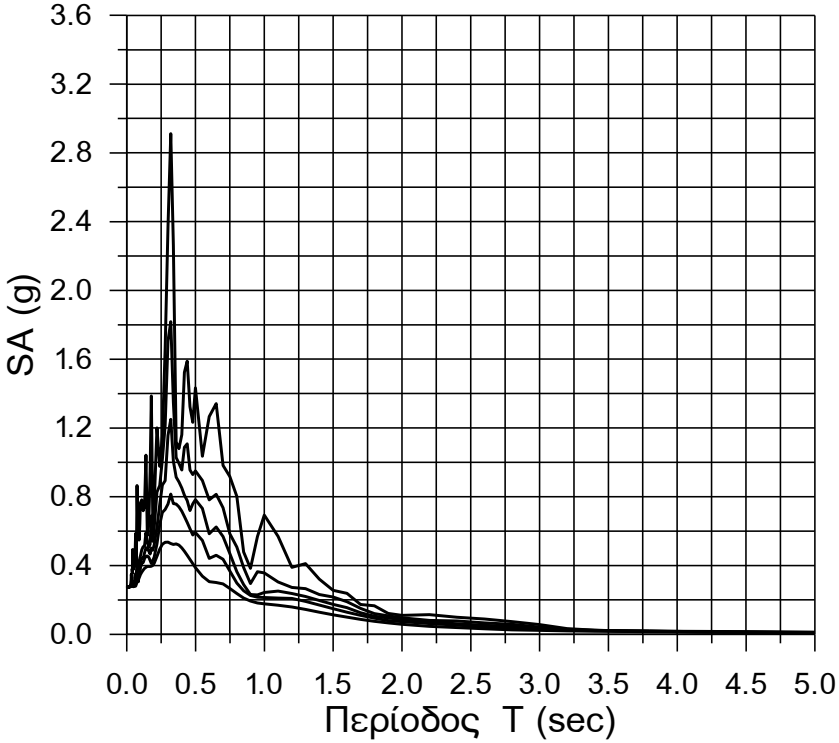


Response spectrum

Kalamata, 1985 earthquake

Response spectra for ζ = 0, 2%, 5%, 10%, 20%





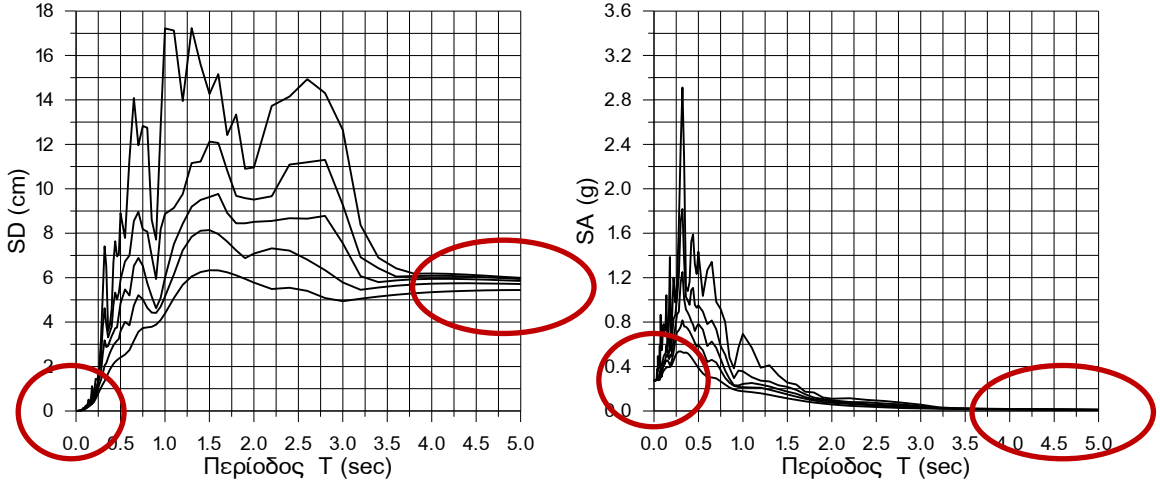
Pseudo-Spectra

For small values of damping ($\zeta \leq 20\%$)

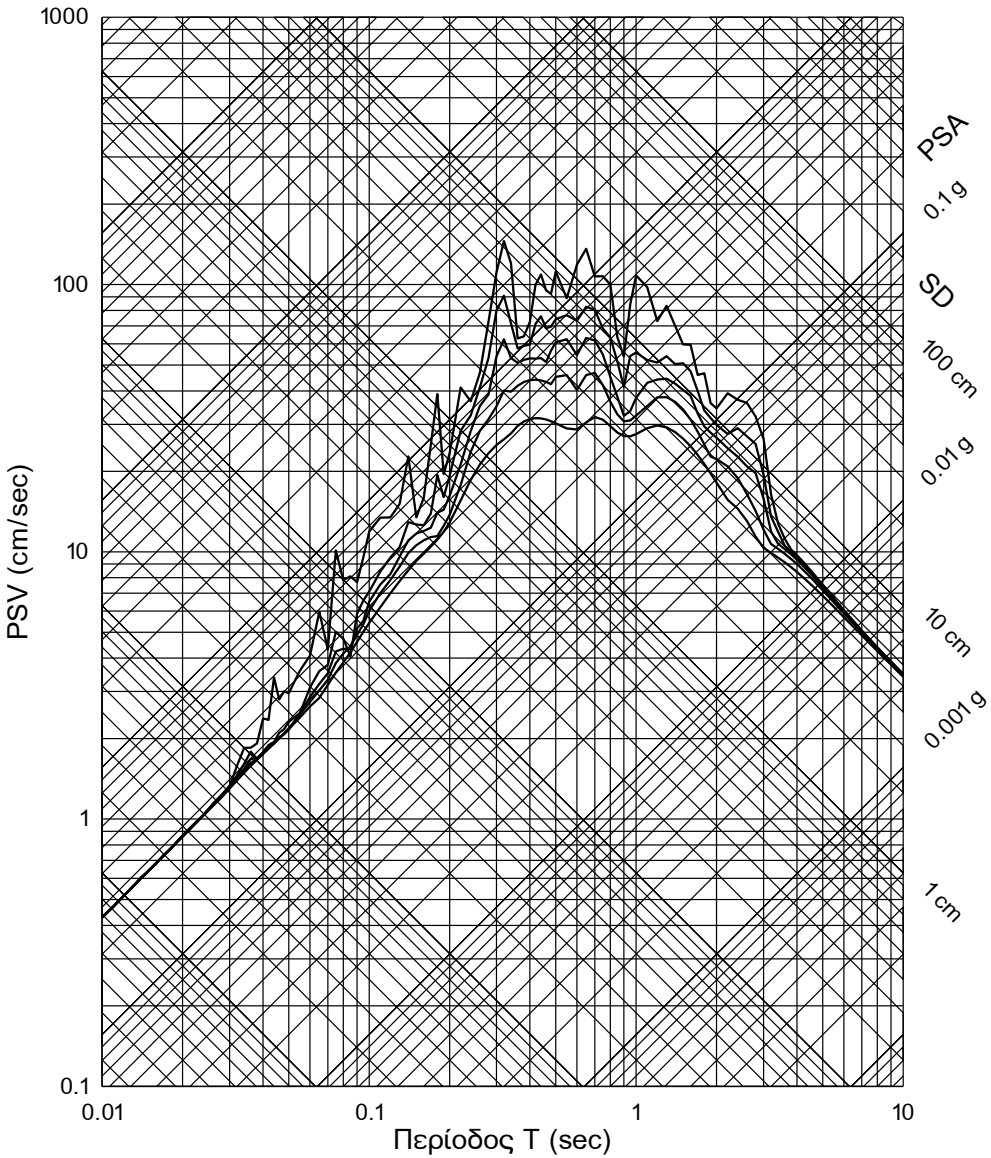
- $SA \cong \omega^2 \cdot SD = PSA$
- $SV \cong \omega \cdot SD = PSV$

Limits:

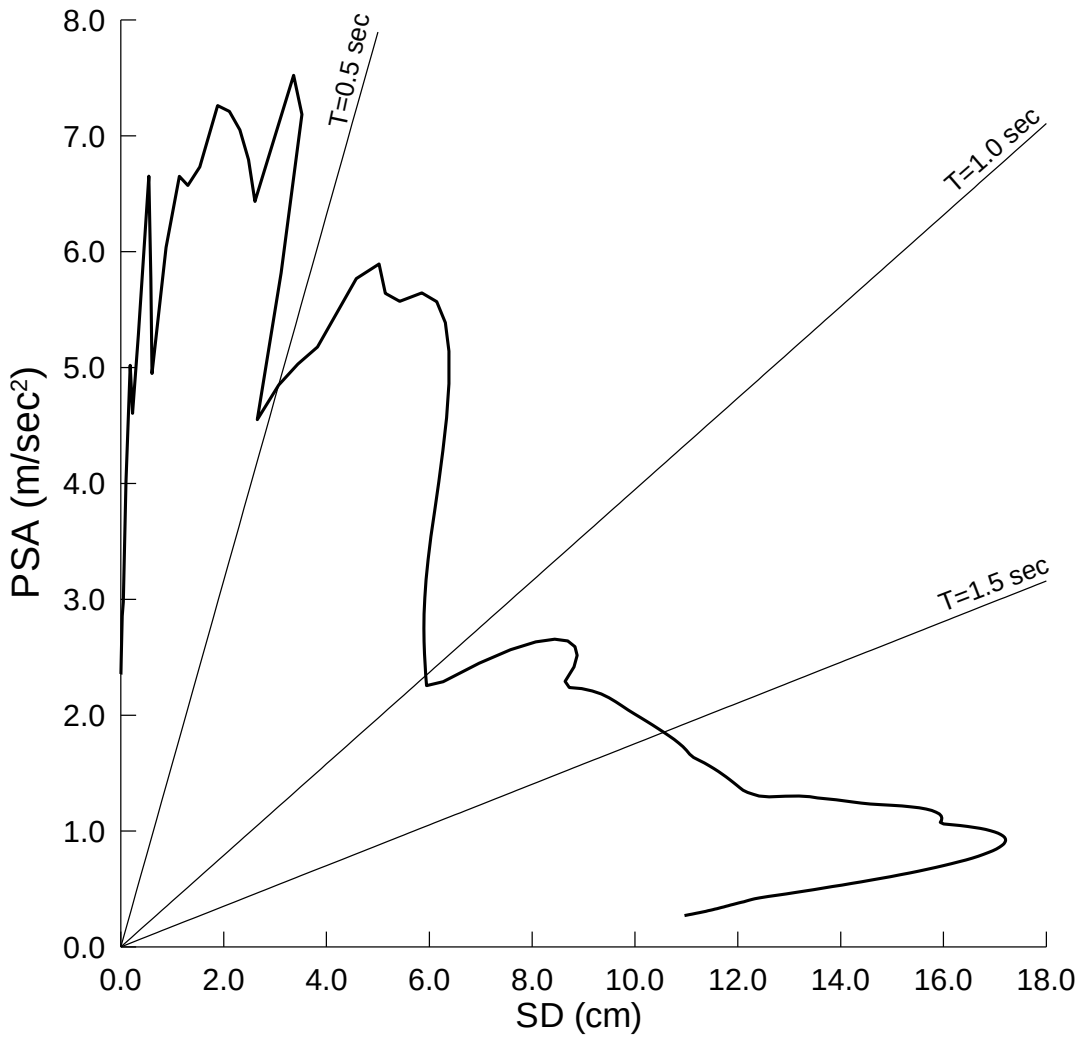
- $T \to 0$: $SD \to 0$ $SV \to 0$ $SA \to \ddot{x}_{g,max}$
- $T \to \infty$: $SD \to x_{g,max}$ $SV \to \dot{x}_{g,max}$ $SA \to 0$



Logarithmic representation

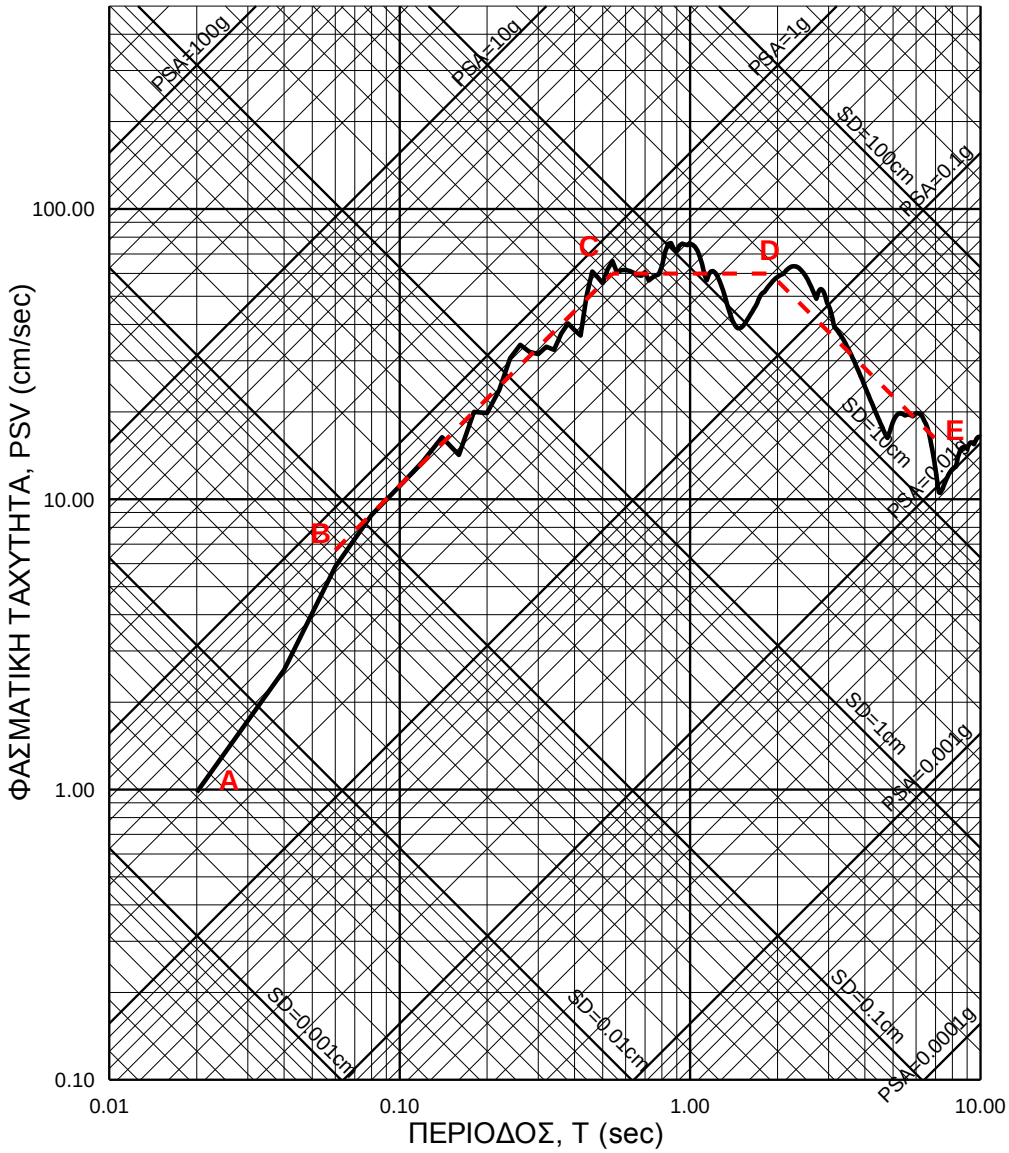


ADRS - representation



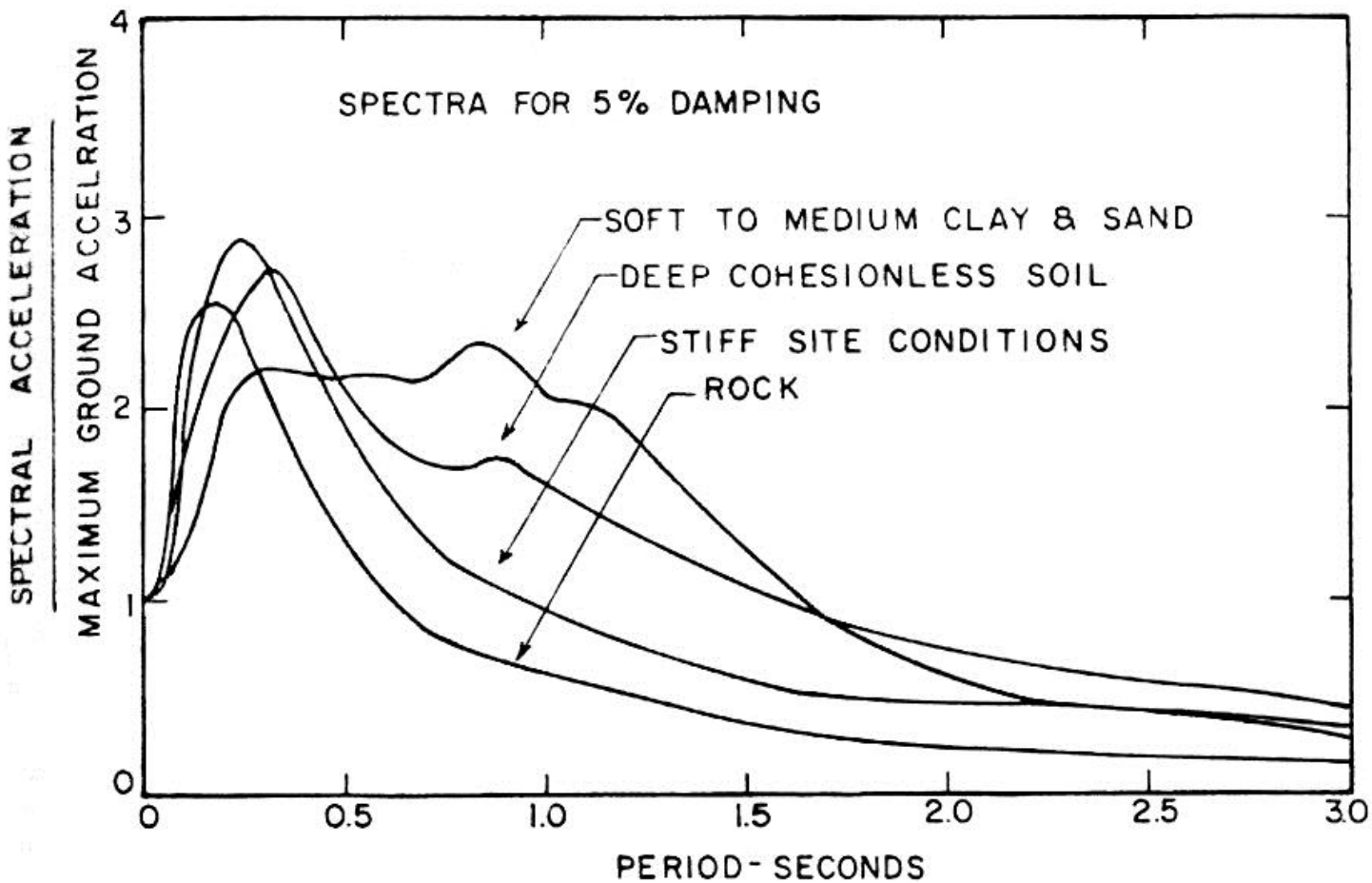
$$T = 2\pi\sqrt{\frac{SD}{PSA}}$$

Characteristic regions



Design spectrum

Effect of soil conditions



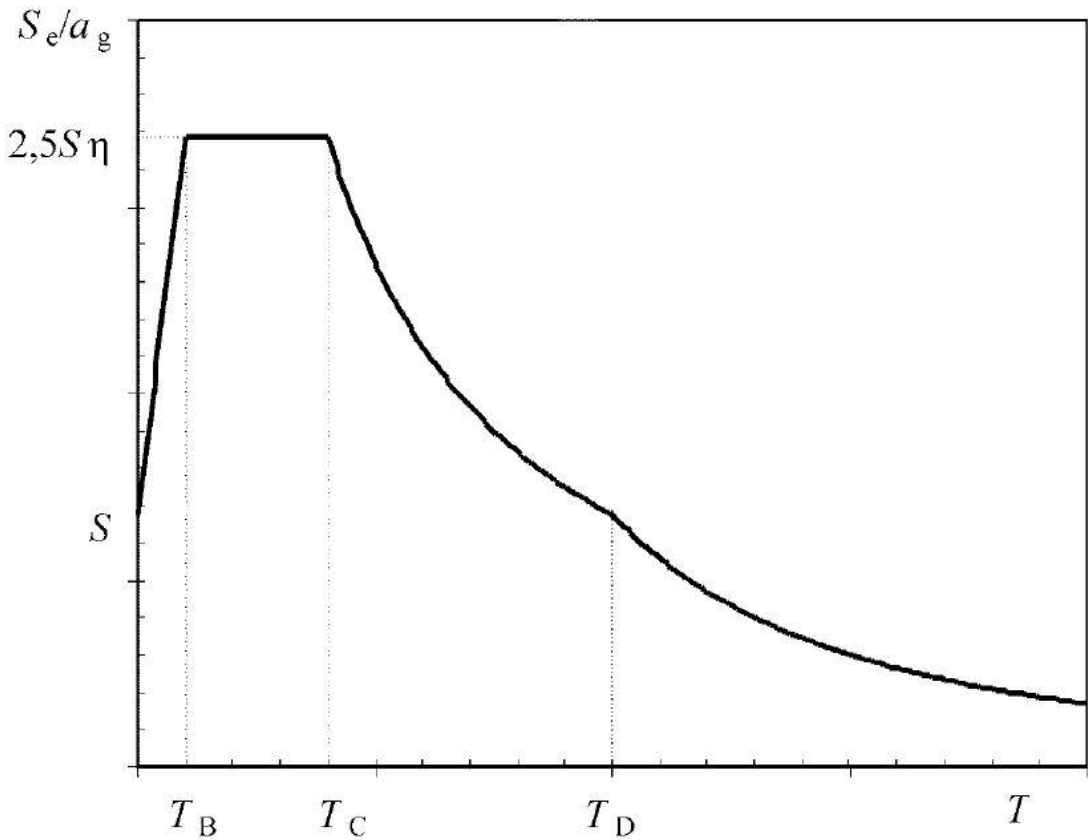
Design spectrum

Ground types according to EC 8

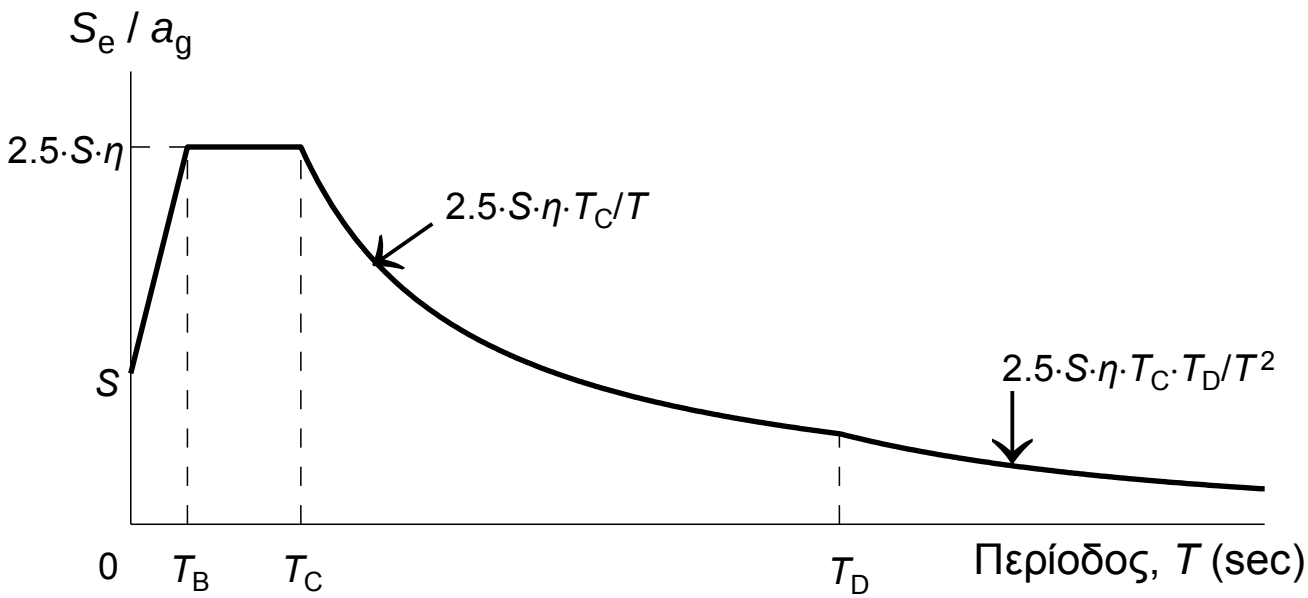
Ground type	Description of stratigraphic profile	Parameters		
		$v_{s,30}$ (m/s)	N_{SPT} (blows/30cm)	c_u (kPa)
A	Rock or other rock-like geological formation, including at most 5 m of weaker material at the surface.	> 800	–	–
B	Deposits of very dense sand, gravel, or very stiff clay, at least several tens of metres in thickness, characterised by a gradual increase of mechanical properties with depth.	360 – 800	> 50	> 250
C	Deep deposits of dense or medium-dense sand, gravel or stiff clay with thickness from several tens to many hundreds of metres.	180 – 360	15 - 50	70 - 250
D	Deposits of loose-to-medium cohesionless soil (with or without some soft cohesive layers), or of predominantly soft-to-firm cohesive soil.	< 180	< 15	< 70
E	A soil profile consisting of a surface alluvium layer with v_s values of type C or D and thickness varying between about 5 m and 20 m, underlain by stiffer material with v_s > 800 m/s.			
S_1	Deposits consisting, or containing a layer at least 10 m thick, of soft clays/silts with a high plasticity index (PI > 40) and high water content	< 100 (indicative)	–	10 - 20
S_2	Deposits of liquefiable soils, of sensitive clays, or any other soil profile not included in types A – E or S_1			

EC 8 – Elastic response spectrum

- $S_e(T)$ = spectral acceleration for period T
- $a_g = \gamma_I a_{gR}$ = design ground acceleration
- γ_I = importance factor
- a_{gR} = reference peak ground acceleration on type A ground
- S = soil factor
- $\eta = \sqrt{\frac{0.10}{0.05+\zeta}}$



EC 8 – Elastic response spectrum



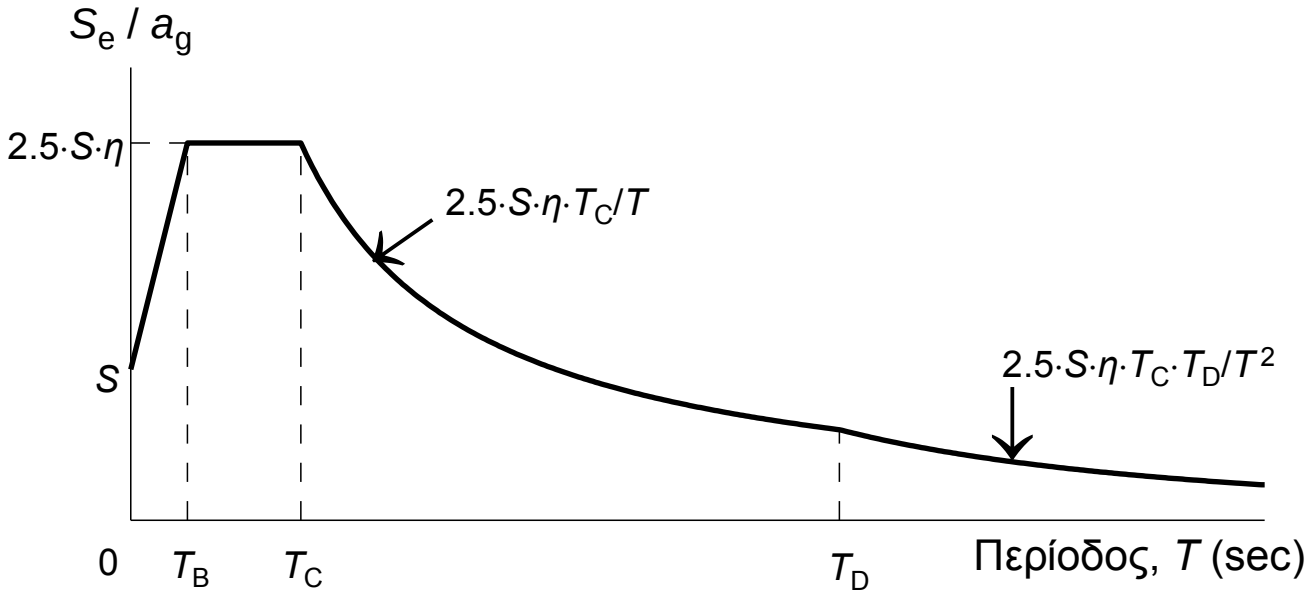
$$S_e(T) = a_g \cdot S \cdot \left[1 + \frac{T}{T_B} \cdot (\eta \cdot 2.5 - 1)\right]$$ for $0 \le T \le T_B$

$$S_e(T) = a_g \cdot S \cdot \eta \cdot 2.5$$ for $T_B \le T \le T_C$

$$S_e(T) = a_g \cdot S \cdot \eta \cdot 2.5 \cdot \frac{T_C}{T}$$ for $T_C \le T \le T_D$

$$S_e(T) = a_g \cdot S \cdot \eta \cdot 2.5 \cdot \frac{T_C \cdot T_D}{T^2}$$ for $T_D \le T \le 4.0$ sec

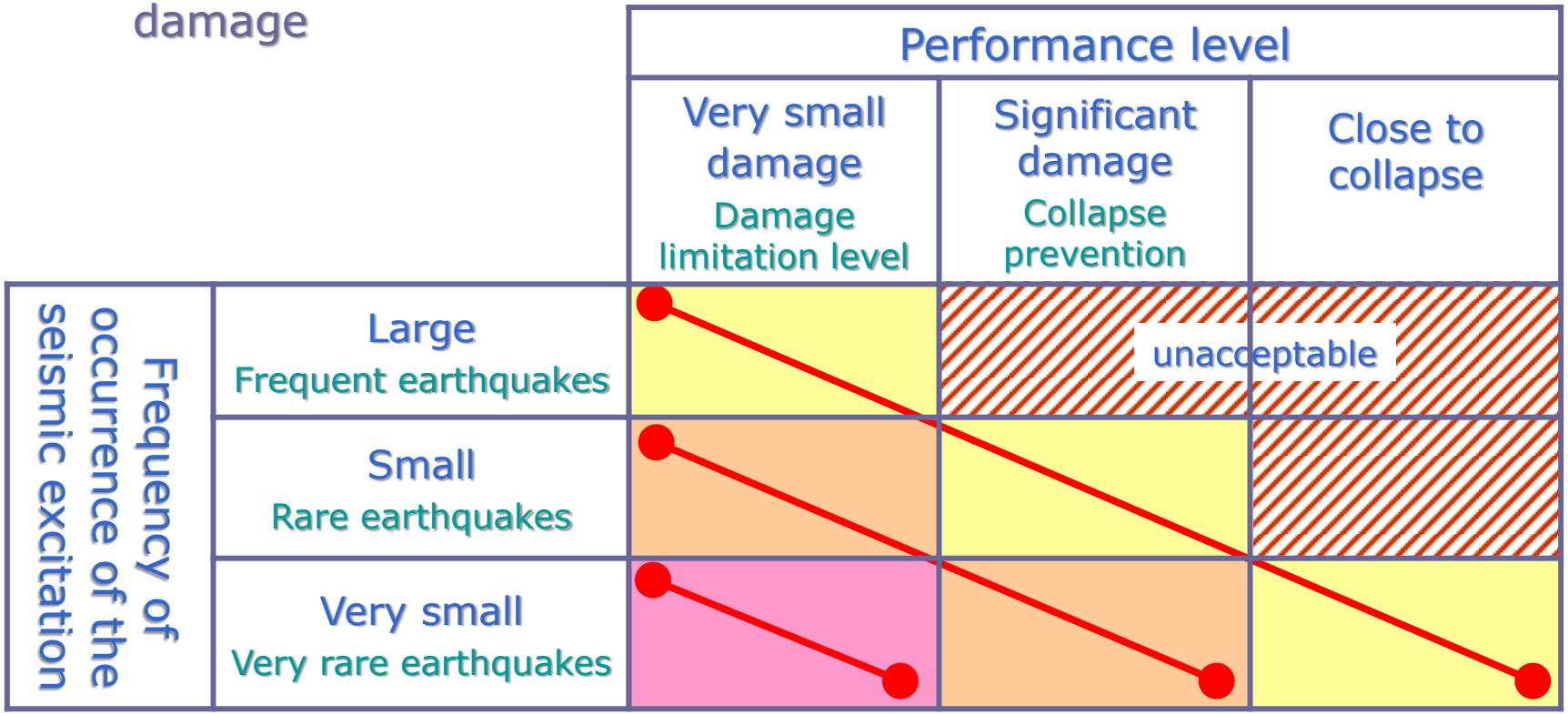
EC 8 – Elastic response spectrum



Ground type	T_B (sec)	T_C (sec)	T_D (sec)	S
A	0.15	0.40	2.50	1.00
B	0.15	0.50	2.50	1.20
C	0.20	0.60	2.50	1.15
D	0.20	0.80	2.50	1.35
E	0.15	0.50	2.50	1.40

Performance levels

- Define earthquake loadings with various probabilities of occurrence
- Define various acceptable level of damage
- Combine each earthquake loading with an acceptable level of damage



Performance requirements of seismic codes

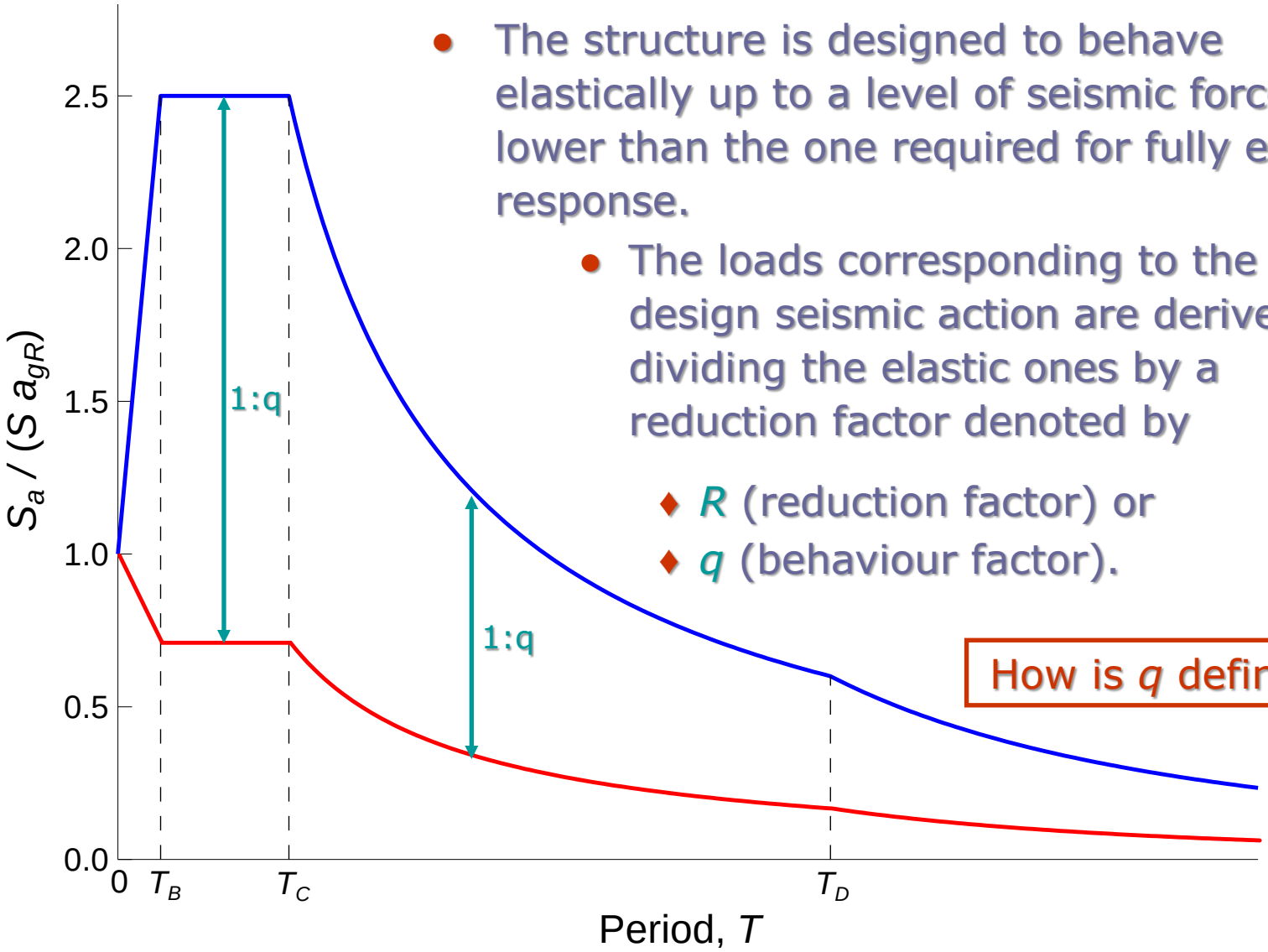
Most codes are based on two performance levels:

- Damage limitation level
  - The structure is expected to experience limited structural and non-structural damage during frequent earthquakes. In this limit state:
    - The structural members retain their strength and stiffness.
    - No permanent deformations and drifts occur.
    - No repair is needed.
  - The seismic action is usually termed as the serviceability earthquake. Reasonable probability of exceedance = 10% in 10 years (mean return period = 95 years).
  - Compliance criteria are usually expressed in terms of deformation limits.

Performance requirements of seismic codes

- Collapse prevention level
  - Ensure prevention of collapse and retention of structural integrity for an earthquake with a small possibility of occurrence during the life of the structure:
    - Significant damage might happen.
    - The structure should be able to bear the vertical loads and retain sufficient lateral strength and stiffness to protect life during aftershocks.
  - The seismic action is referred as the design earthquake. For structures of ordinary importance: 10% probability of exceedance in 50 years (mean return period of 475 years).
  - Compliance criteria are expressed in terms of forces (force-based seismic design).

Design concept



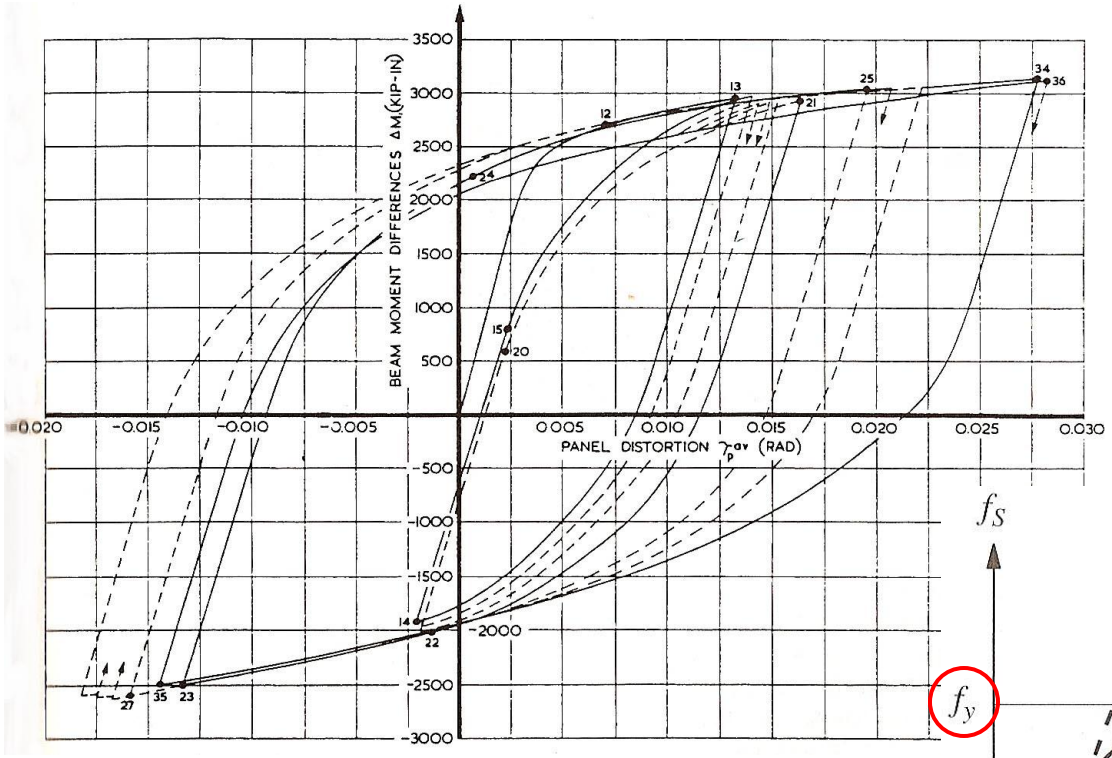
- The structure is designed to behave elastically up to a level of seismic forces lower than the one required for fully elastic response.
  - The loads corresponding to the design seismic action are derived by dividing the elastic ones by a reduction factor denoted by
    - *R* (reduction factor) or
    - *q* (behaviour factor).

How is *q* defined?

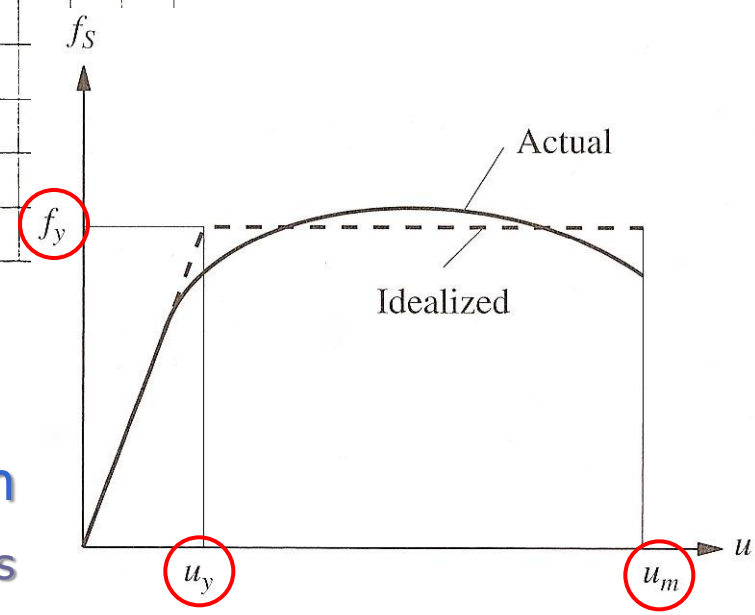
Ductility capacity

- The collapse mechanism of the structural members is related to their deformation and not to the forces induced to them during the seismic action.
- In order to comply with the non-collapse criterion, an overall ductile behaviour should be ensured.
- In other words: the structure should have an adequate capacity to deform beyond its elastic limit without substantial reduction in the overall resistance against horizontal and vertical loads.
- This is achieved through proper dimensioning and detailing of the structural elements.
- In addition, capacity design concepts are applied, in order to ensure that ductile modes of failure (e.g. flexure) should precede brittle modes of failure (e.g. shear) with sufficient reliability.

Nonlinear response



Force-deformation relation



Elastoplastic idealization

Same area under the two curves

Basic definitions

- Yield strength behaviour factor:

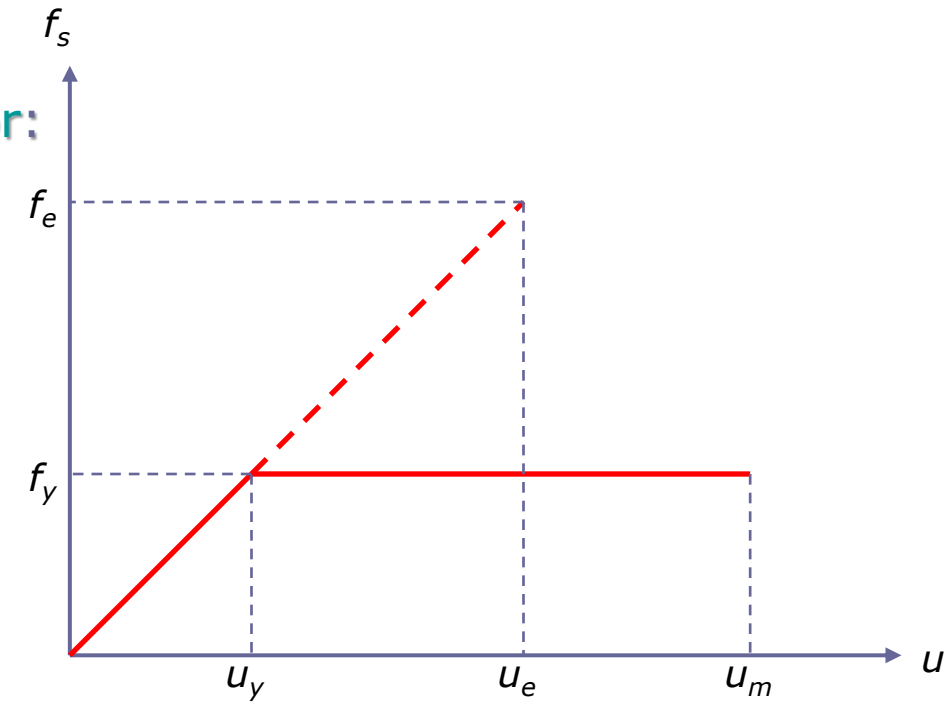
$$q_y = \frac{f_e}{f_y}$$

- Ductility factor:

$$\mu = \frac{u_m}{u_y}$$

- It can easily be proved:

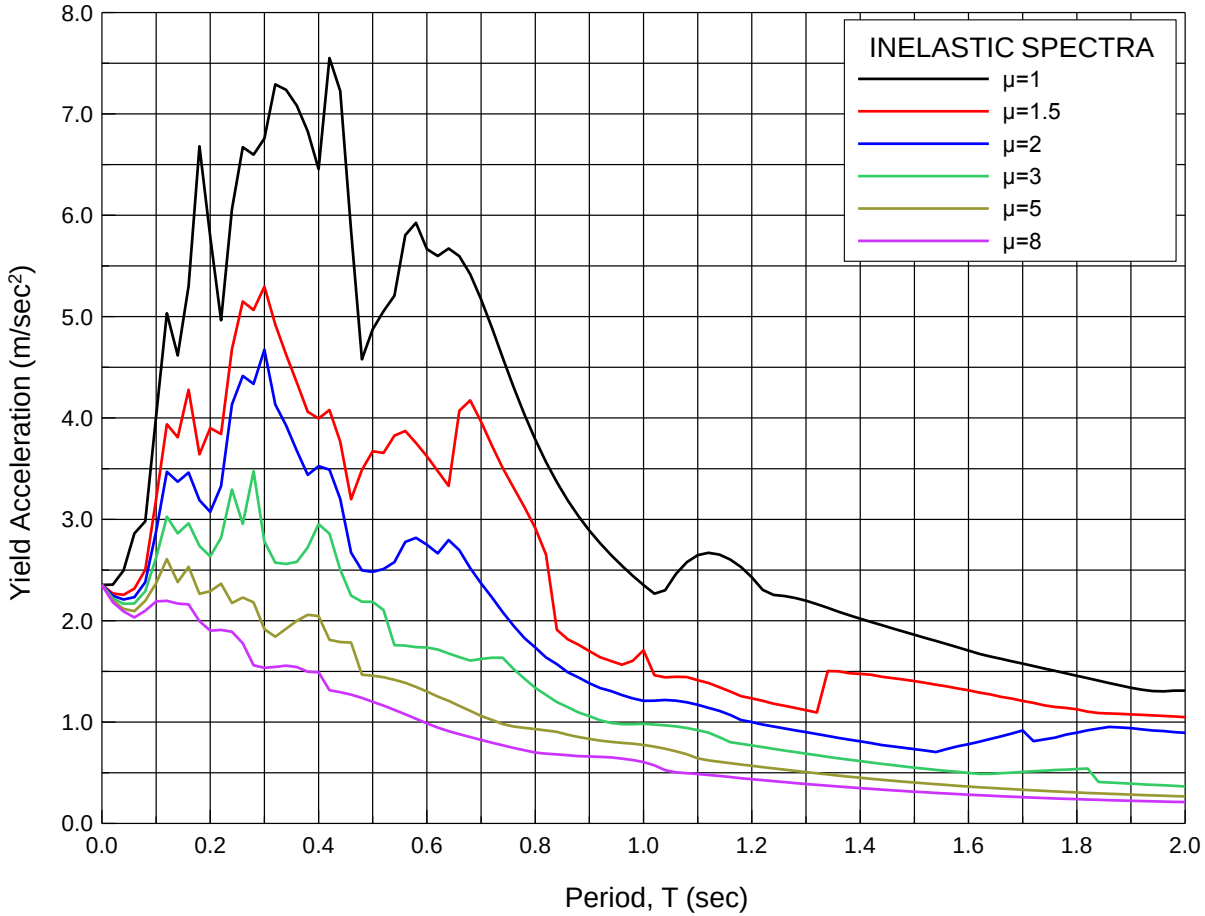
$$\frac{u_m}{u_e} = \frac{\mu}{q_y}$$



- Larger values of μ correspond to larger plastic deformation $\Rightarrow$ more damage.
- For μ close to 1, the response is close to the elastic.

Inelastic response spectra

Inelastic spectra for constant ductility



Ductility factor

- The damage that will be induced to the structure is directly related to the ductility factor, μ.
- For the non-collapse performance criterion, certain values can be assigned to the allowable maximum value of μ, depending on:
  - The material (ductile or brittle).
  - The structural system (the more isostatic is the structure the less is the allowable value of μ).
  - The structural irregularities in plan or in elevation and the torsional sensitivity (reduce the allowable value of μ).
  - The connections and the bracing types (steel structures).

Relations q_y - μ

- For $T>T_C$ the equal displacement assumption is made:

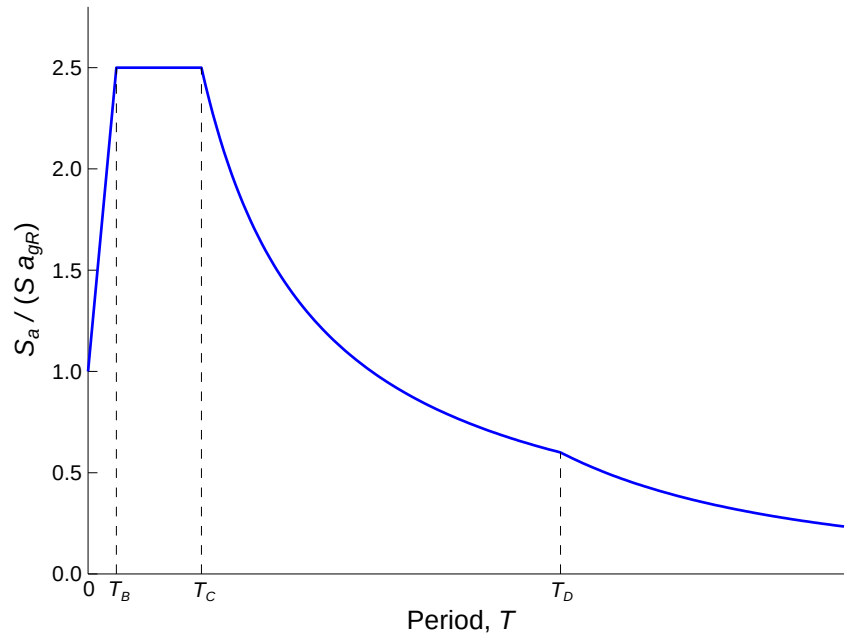
$$q_y = \mu$$

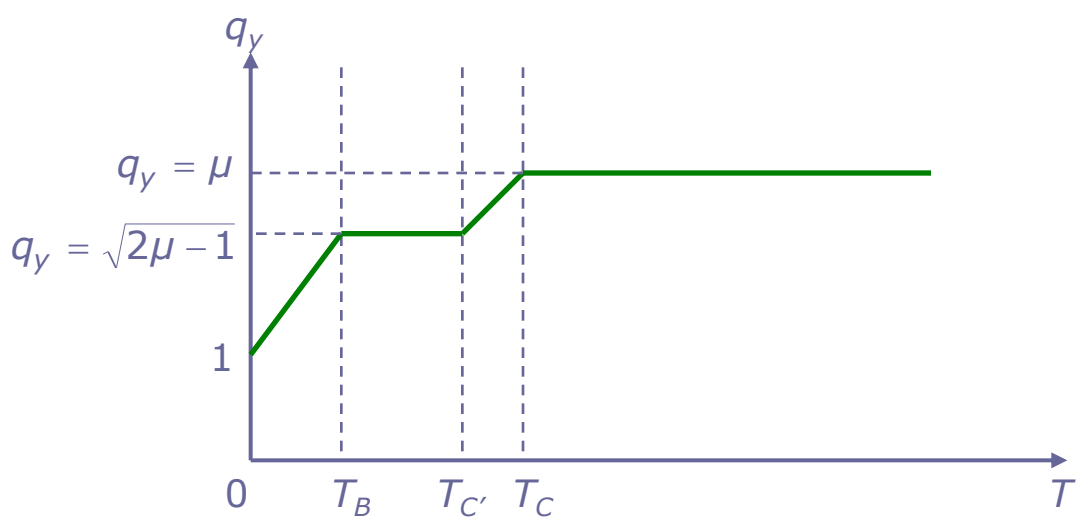
- For $T_B<T<T_{C'}$ the equal energy assumption is made:

$$q_y = \sqrt{2\mu - 1}$$

- For T close to zero (very stiff structures) the response is elastic:

$$q_y = 1$$





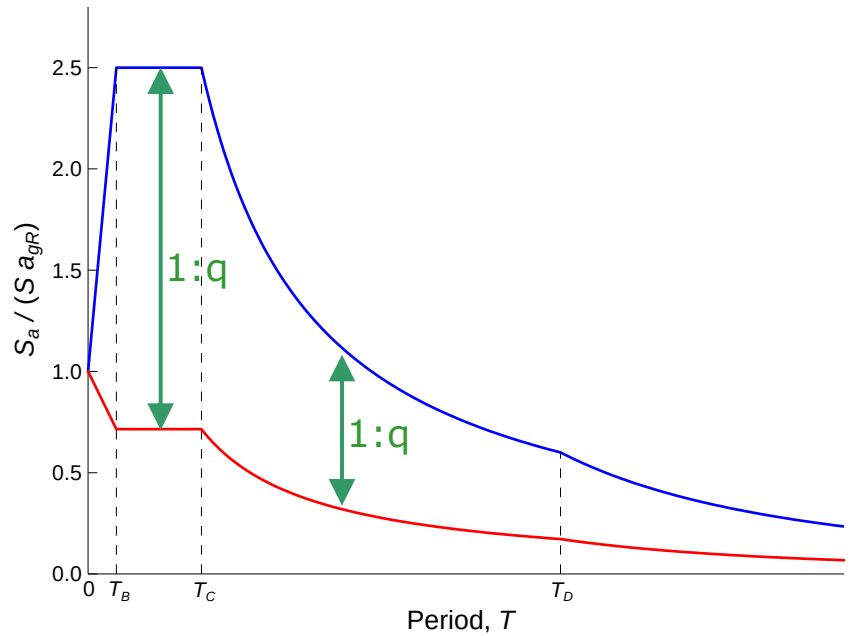
Design value of q

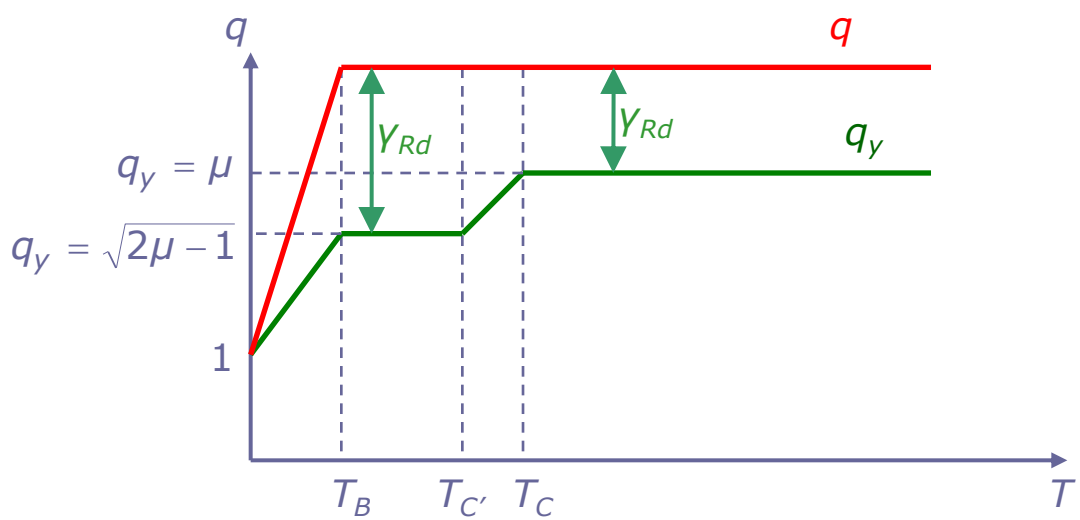
Design value of the behaviour factor:

$$q = \gamma_{Rd} \cdot q_y$$

(γ_{Rd}= overstrength)

- Usually, rigid structures possess larger overstrength than flexible ones ⇒ we usually assume constant value of q for $T > T_B$.





Ductility classes (EC8)

- Ductility Class High (DCH)
  - Strict detailing criteria should be fulfilled.
  - Provides higher safety margins against local or global collapse under seismic actions stronger than the design earthquake.
- Ductility Class Medium (DCM)
  - Compared to DCH, certain detailing rules are relaxed.
  - The design leads to slightly easier to construct structures.
  - Provides good performance during moderate earthquakes.
- Ductility Class Low (DCL)
  - For low seismicity areas.
  - The structure is designed according to EC2 without special seismic considerations.
  - Large values of q are allowed.

Proper detailing

Aims to:

- provide the structure with an adequate capacity to deform beyond its elastic limit without substantial reduction of the overall resistance against horizontal and vertical loads.

Example for concrete structures:

Special rules are applied for the confinement reinforcement (stirrups) at column-to-beam joints and at critical regions of columns and beams.

Capacity design

Aims to:

- ensure that ductile modes of failure (e.g. flexure) should precede brittle modes of failure (e.g. shear) with sufficient reliability
- prevent the formation of a soft-story mechanism
- ensure that certain parts of the structure will remain elastic if it is so desired (e.g. foundation, bridge deck, etc.)

Example for concrete structures:

At column-to-beam joints, the sum of the design values of the moments of resistance of the columns should be larger than 1.3×the sum of the design values of the moments of resistance of the beams:

$$\sum M_{Rc} \geq 1.3 \cdot \sum M_{Rb}$$

Design procedure

- Define the seismic loads for:
  - The appropriate seismicity, the soil conditions at the site and the importance of the structure.
  - The appropriate value of the behaviour factor, *q*
    - Material
    - Structural system
    - Irregularites
    - Ductility class
- Perform a structural analysis of the structure for the seismic and non-seismic loads, assuming elastic response.
- Combine the individual load cases according to the code provisions to get the envelop of the member loads.

Design procedure (cont'd)

- Perform the dimensioning of the beams in flexure.
- Check the beams in shear using the capacity design approach (based on the flexural strength of the beams).
- Perform the dimensioning of the columns in flexure using the capacity design approach (based on the flexural strength of the beams framing with the columns at the joints).
- Check columns in shear using the capacity design approach (based on the flexural strength of the columns).
- Perform a detailed dimensioning of the joints in order to assure their integrity during the design earthquake.
- Perform the dimensioning of the foundation using the capacity design approach (based on the flexural strength of the columns).
- Design displacements: $d = q \cdot d_E$, d_E = from seismic analysis.

MDOF systems

Equation of motion

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = [M]\{r\}\ddot{x}_g(t)$$

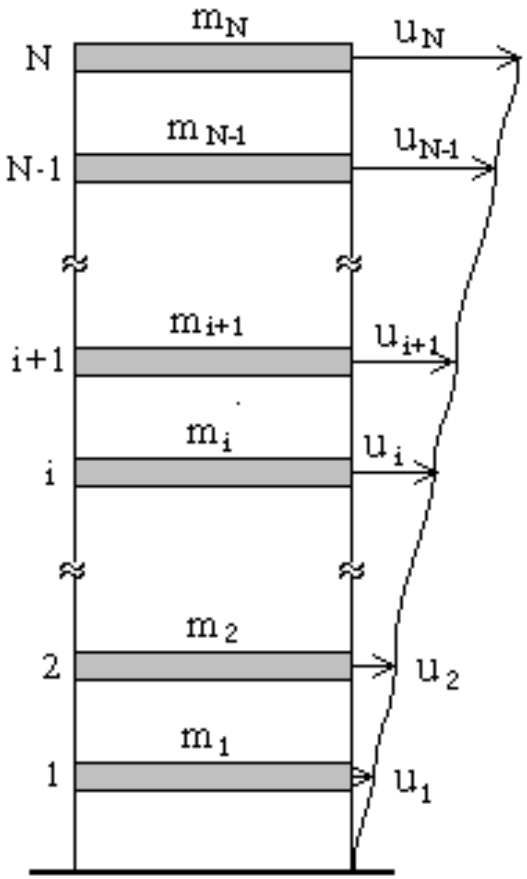
where:

$[M]$ = mass matrix

$[C]$ = damping matrix

$[K]$ = stiffness matrix

$\{r\}$ = earthquake direction vector



Natural modes

- Eigenfrequencies

  They are derived from the solution of the characteristic equation:

  $$|[K] - \omega^2[M]| = 0$$

- Eigenmodes (eigenvectors, eigenshapes)

  They are derived from the solution of the system of equations:

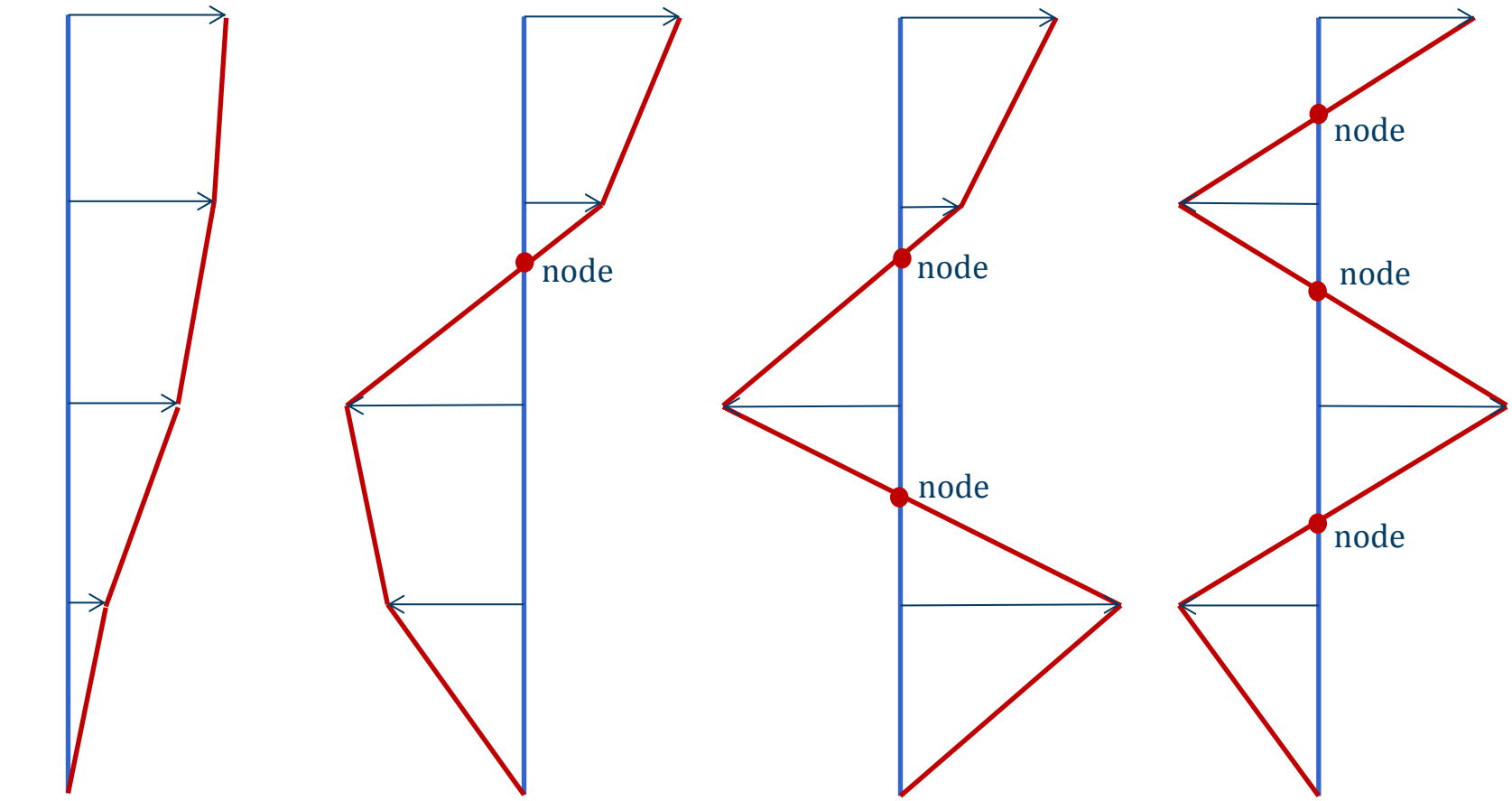
  $$([K] - \omega^2[M])\{\varphi_i\} = \{0\}$$

  where:

  $\{\varphi_i\}$ = i^{th} eigenmode

  φ_{ji}= j^{th} component of ith eigenmode

Eigenmodes



1st mode 2nd mode 3rd mode 4th mode

Properties

- Orthogonality

  $\{\varphi_i\}^{\mathrm{T}}[M]\{\varphi_j\} = 0 \quad \text{for } i \neq j$

  $\{\varphi_i\}^{\mathrm{T}}[K]\{\varphi_j\} = 0 \quad \text{for } i \neq j$

- Generalized mass

  $\tilde{m}_i = \{\varphi_i\}^{\mathrm{T}}[M]\{\varphi_i\}$

- Generalized stiffness

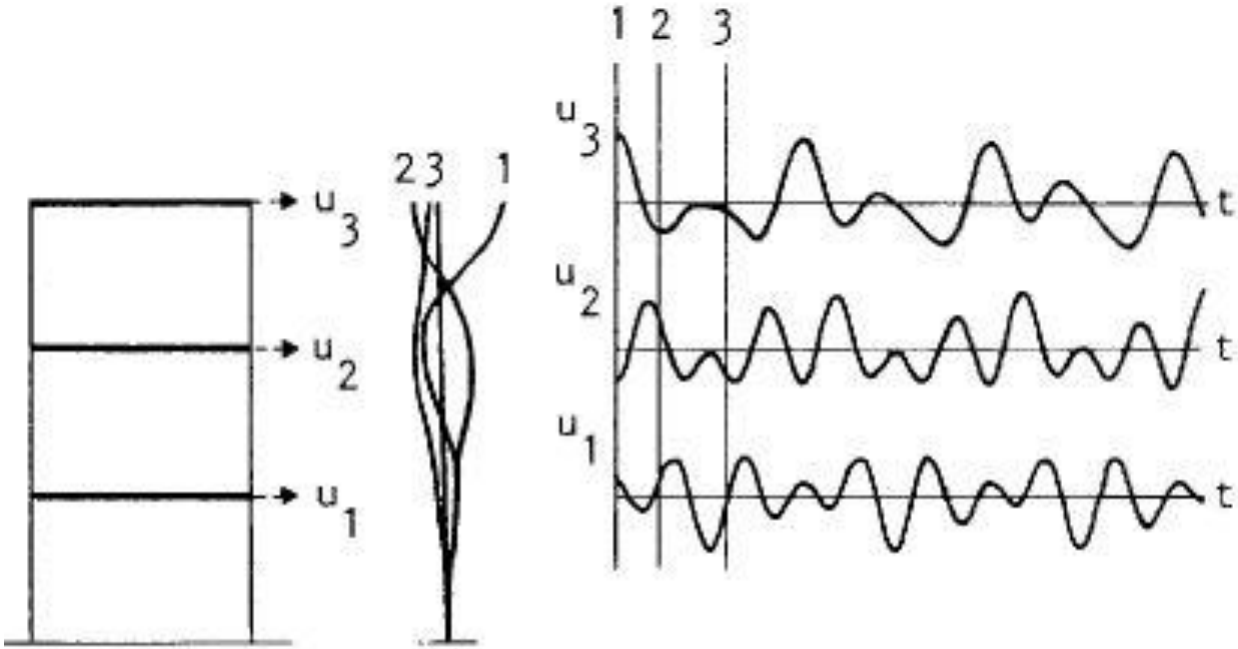
  $\tilde{k}_i = \{\varphi_i\}^{\mathrm{T}}[K]\{\varphi_i\}$

- It can be shown that

  $\tilde{k}_i = \tilde{m}_i \cdot \omega^2$

Free vibrations

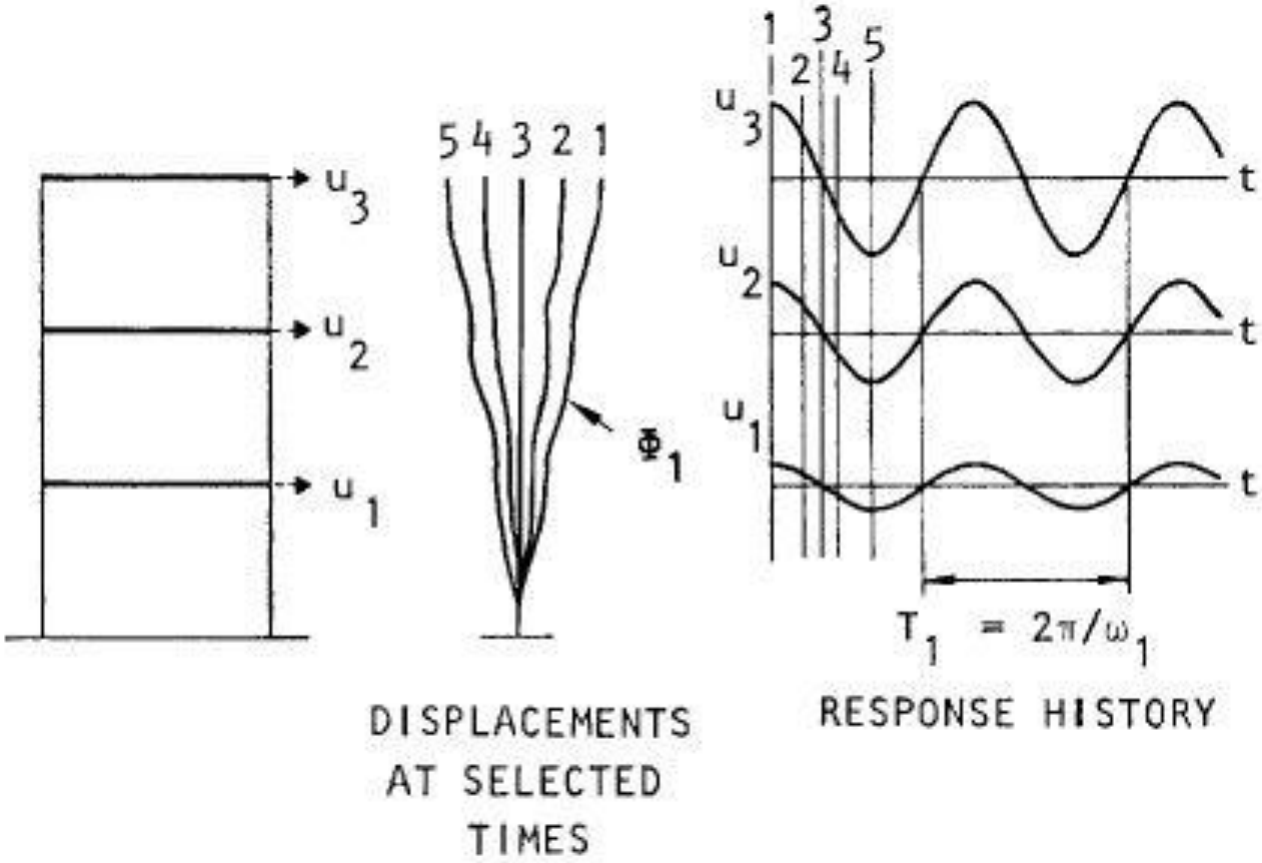
For arbitrary initial displacements



DEFORMED
POSITIONS
AT TIME INSTANTS
1, 2 AND 3

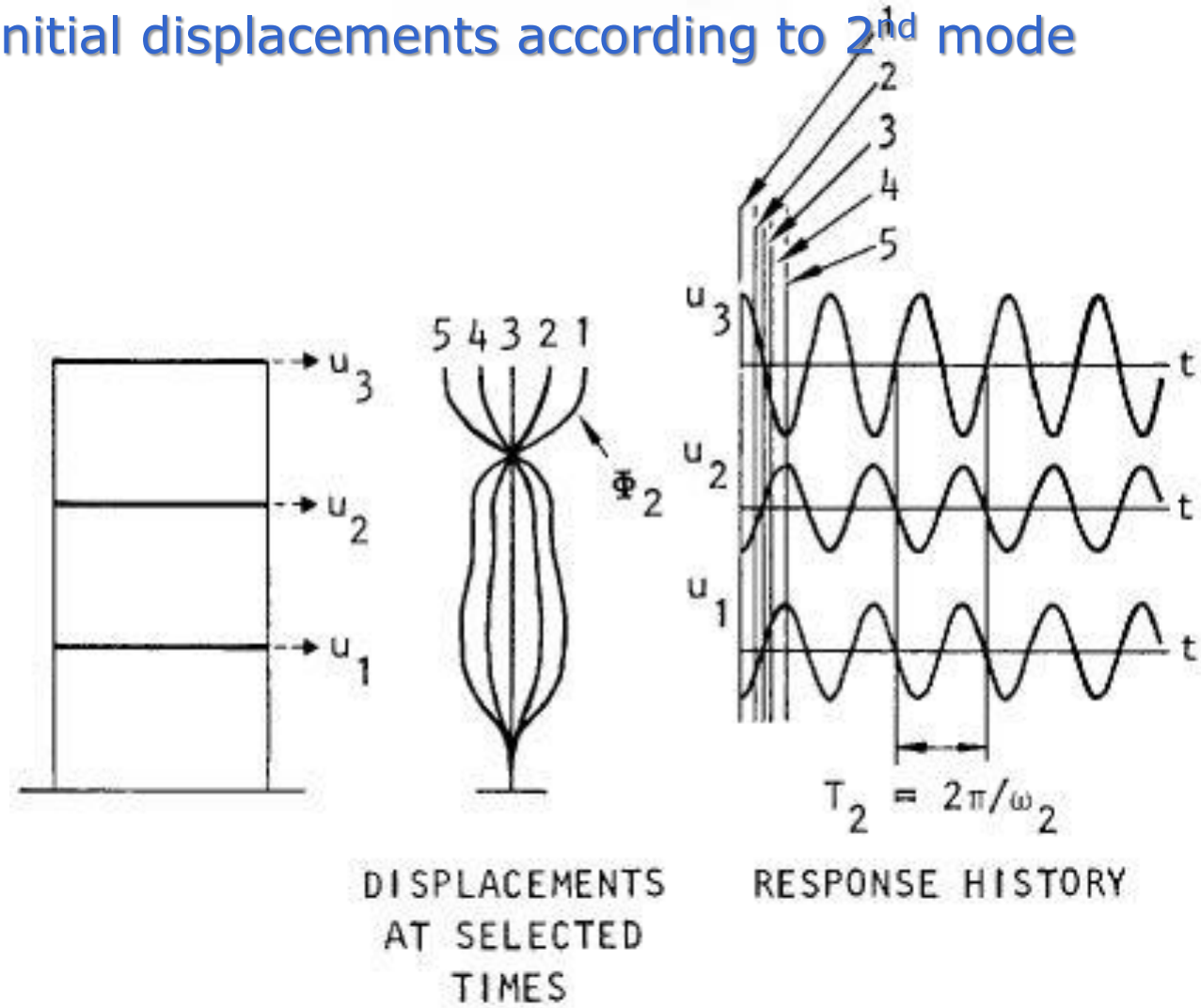
Free vibrations

For initial displacements according to 1st mode



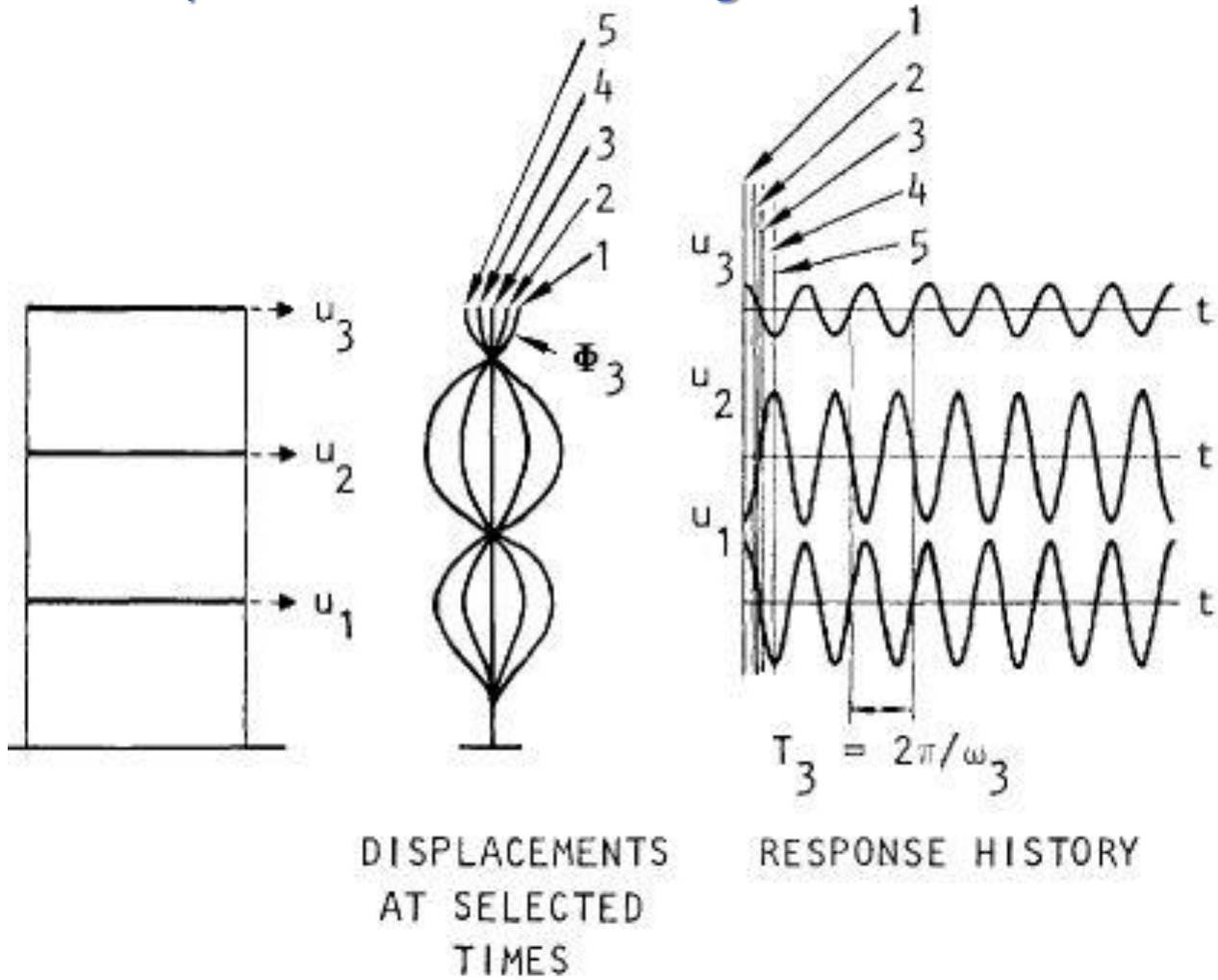
Free vibrations

For initial displacements according to 2nd mode



Free vibrations

For initial displacements according to 3rd mode



Modal analysis

- Displacement at the jth degree of freedom:

$$u_j(t) = \sum_{n=1}^{N} u_{jn}(t)$$

where u_{jn} is the displacement of the jth degree of freedom that corresponds to the nth mode.

- Response of nth mode:

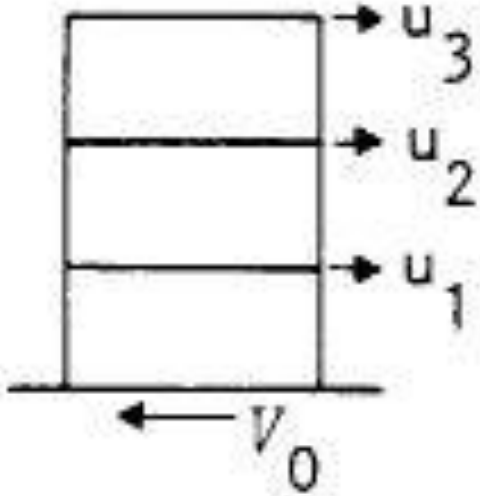
$$u_{jn}(t) = Y_n(t) \cdot \varphi_{jn}$$

$$\ddot{Y}_n + 2\zeta_n \omega_n Y_n + {\omega_n}^2 Y_n = \Gamma_n \ddot{x}_g$$

where Γ_n is the participation factor of the nth mode:

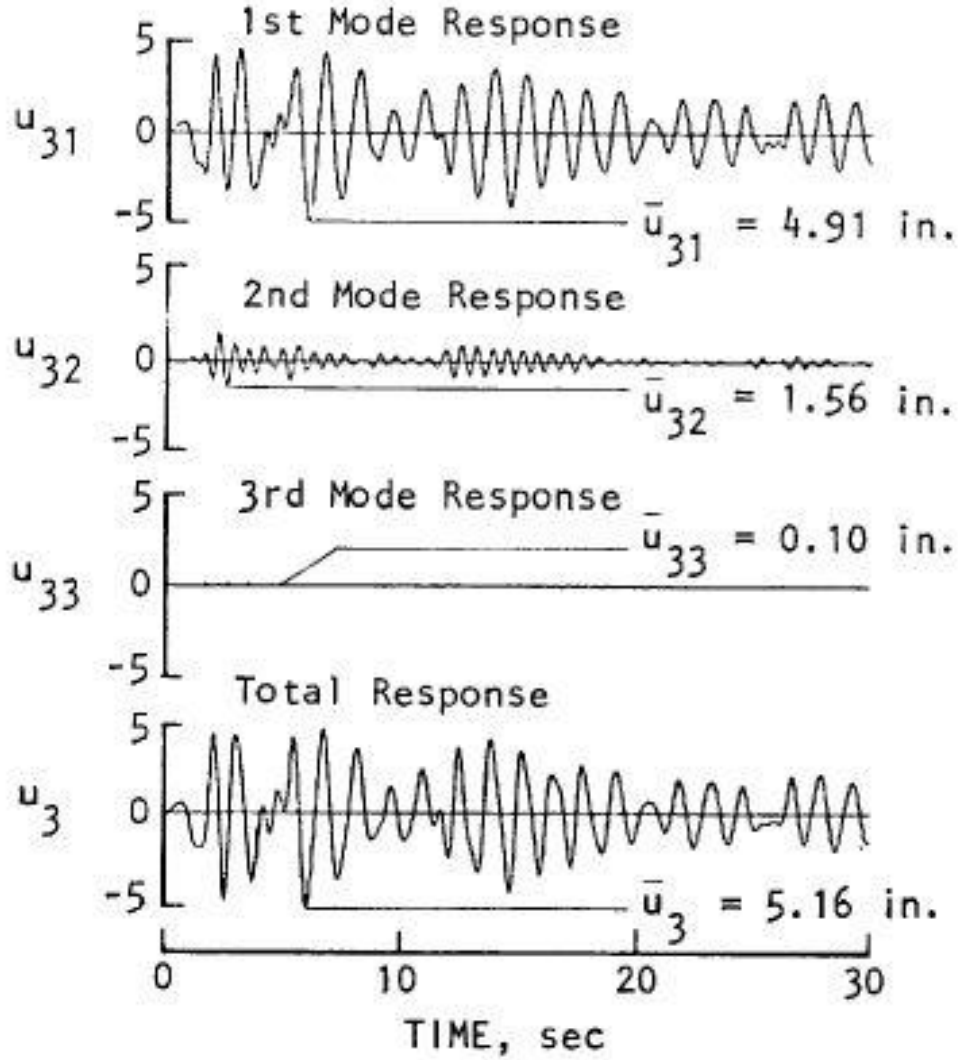
$$\Gamma_n = \frac{\{\varphi_i\}^{\mathrm{T}}[M]\{r\}}{\{\varphi_i\}^{\mathrm{T}}[M]\{\varphi_i\}}$$

Modal analysis



$$u_{jn}(t) = Y_n(t) \cdot \varphi_{jn}$$

$$\ddot{Y}_n + 2\zeta_n \omega_n Y_n + \omega_n{}^2 Y_n = \Gamma_n \ddot{x}_g$$



Use of response spectra

- Maximum displacement of the nth mode at the jth degree of freedom:

$$max|u_{jn}| = \Gamma_n \cdot S_d(T_n, \zeta_n) \cdot \varphi_{jn}$$

where $S_d(T_n, \zeta_n)$ is the spectral displacement that corresponds to period T_n and damping ζ_n.

- Maximum seismic force of the nth mode at the jth degree of freedom:

$$max|F_{jn}| = \Gamma_n \cdot S_{a,d}(T_n, \zeta_n) \cdot m_j \cdot \varphi_{jn}$$

where $S_{a,d}(T_n, \zeta_n)$ is the design spectral acceleration that corresponds to period T_n and damping ζ_n.

Combination of modal responses

- Significant modes

$$k < N$$

$$\sum_{n=1}^{k} M_n \geq 0.90 \cdot m_{tot}$$

where M_n is the effective mass of the nth mode:

$$M_n = \Gamma_n \cdot \{\varphi_n\}^{\mathrm{T}}[M]\{r\}$$

The effective mass of each mode depends on the direction of the seismic action.

Combination of modal responses

Let A_n, A_m be the maximum value of a quantity A (internal force or displacement) of the nth and the mth mode respectively.

- SRSS

$$maxA = \pm\sqrt{\sum_{n=1}^{k} {A_n}^2}$$

- CQC

$$maxA = \pm\sqrt{\sum_{n=1}^{k}\sum_{m=1}^{k} \varepsilon_{nm} A_n A_m}$$

$$\varepsilon_{nm} = \frac{8\cdot\zeta^2\cdot(1+r)\cdot r^{3/2}}{(1-r^2)^2+4\cdot\zeta^2\cdot r\cdot(1+r)^2} \quad \text{with} \quad r = \frac{T_n}{T_m}$$

Simplified formulas

For planar motion in the plane of the seismic action and for the nth mode:

$$\Gamma_n = \frac{\sum_{j=1}^{N} m_j \varphi_{jn}}{\sum_{j=1}^{N} m_j \left(\varphi_{jn}\right)^2}$$

$$M_n = \Gamma_n \cdot \sum_{j=1}^{N} m_j \varphi_{jn}$$

Design procedure

- Define structural properties
  - Compute mass and stiffness matrices $[M]$ and $[K]$
  - Estimate modal damping coefficients ζ_n
- Solve the eigen-problem to determine the *k* lower natural frequencies ω_n and modes $\{\varphi_n\}$, $1 \leq n \leq k$
- Compute the corresponding natural periods $T_n = 2\pi/\omega_n$
- For a given direction of the seismic action:
  - Compute the participation factors Γ_n
  - Compute effective modal masses M_n and check that their sum is larger than 90% of the total mass. If not, increase the value of *k* and repeat the procedure

Design procedure

- For a given direction of the seismic action, compute the maximum response for each mode *n* by repeating the following steps:
  - On the design response spectrum read the spectral acceleration $S_{a,d}$ that corresponds to period T_n and damping ζ_n
  - Compute the seismic force $F_{j,n}$ at each degree of freedom *j*
  - Perform static analysis of the structure subjected to forces $F_{j,n}$ and determine the modal internal forces and displacements
- Combine the modal responses using SRSS or CQC for each direction of the seismic action

Spatial combination

Let A_x, A_y, A_z be the estimated maximum values of a quantity A that correspond to two horizontal orthogonal directions x, y and the vertical direction z of the seismic action.

The maximum value of A for simultaneous action of the earthquake in all directions x, y, z can be estimated as:

- 1st Method

$$A = \pm\sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$$

- 2nd Method

$$A = \pm A_x \pm 0.3A_y \pm 0.3A_z \quad \text{or}$$

$$A = \pm 0.3A_x \pm A_y \pm 0.3A_z \quad \text{or}$$

$$A = \pm 0.3A_x \pm 0.3A_y \pm A_z$$

Remarks

- For typical buildings, the vertical component of the seismic action can be neglected.
- Displacements

  If d_E are the displacements from the above analysis, the actual displacements are calculated as

  $$d = q \cdot d_E$$

  where q is the value of the behavior factor used in the design response spectrum.
- The elastic response of a structure to a specific earthquake can also be computed using the above procedure by substituting the design response spectrum with the elastic spectrum of the ground motion.